J. JEAN AJDLER

Luhot Ha-Ibbur Part I: Rabbi Raphael Ha-Levi from Hanover's Tables of Intercalation

Rabbi Raphael Ha-Levi from Hanover is mainly known for his book, *Tekhunat ha-Shamayim*. Although it was published without the author's knowledge from his students' notes, it allows readers to understand the principles of ancient astronomy and explains the principles adopted by Maimonides in his Laws of Sanctifying the New Moon (*Hilkhot Kiddush ha-Hodesh*). However, Ha-Levi's masterpiece is his book *Luhot ha-Ibbur*. This book allows even non-German readers to calculate the true conjunctions and oppositions, and check the occurrence of solar and lunar eclipses. Ha-Levi's intercalation tables are calculated with the highest precision, and are the lasting evidence of his exceptional calculation skills. However, the author did not provide any explanation or justification for using his tables. Except for an initial success, which resulted in a second increased edition under the name *Yirat Shamayim* by Meir Fürth, the book was forgotten. This article explains the meaning of Ha-Levi's various intercalation tables and how they were constructed, and also discusses the tables' accuracy.

BIOGRAPHICAL BACKGROUND AND PUBLICATIONS

Raphael Ha-Levi or Raphael Hanover was born in 1685 in Weikersheim. His parents established themselves in Hanover. In his youth he studied at the Yeshiva of Frankfurt-am-Main, where he received a traditional Talmudic education. Later he worked as a bookkeeper in the banking firm of Simon Wolf Oppenheimer in Hanover, and taught himself mathematics and astronomy. Hanover caught the attention of a civil engineer called Mölling who introduced him to the famous Leibnitz.[1] Hanover became Leibnitz's devoted pupil, studying mathematics,

I thank engineer Eran Raviv who read this article and made some important remarks.

1 Leibnitz was one of the greatest scholars and philosophers of the seventeenth century (1646–1716).

astronomy and natural philosophy[2] under his tutelage. Hanover became also Leibnitz's secretary, friend and collaborator. It seems that Hanover later made a living teaching mathematics and astronomy. Practically all his remnant works, both printed and unprinted, deal with subjects from these two fields.[3]

Hanover's good fame extended not only to the Jewish world in Germany and abroad, but also among the gentiles. During the King of England's 1748 visit to Hanover, Hanover proposed an invention that met with the approval of both the English Admiralty and the Royal Society. He arrived in London during April 1748, at the invitation of these bodies, to clarify certain doubts. Hanover's invention was supposed to enable an easy determination of the longitude of any position of a ship at sea.[4]

When Moses Mendelssohn stayed in Hanover in 1771 and 1777, he visited Ha-Levi. Raphael Ha-Levi, had, despite his great age, preserved his physical and mental robustness, despite many blows of fate: his wife died in 1770 and of his seven children, only one daughter had survived. Hanover wrote the following books:

1. Sefer Tekhunat Ha-Shamayim. Amsterdam, 1756.[5]

2 In contemporary parlance, "natural philosophy" is physics.

3 Except for Ha-Levi's manuscript in the Staatsbibliothek Berlin which deals with the calculation of the date of redemption.

4 This was the great problem of this epoch; see: *Greenwich Time and the Longitude* by Derek Howse, 1997. The name of Raphael Ha-Levi is not mentioned in this book.

5 This book was published in Amsterdam without Ha-Levi's knowledge by Moses of Tiktin who added some of his own explanations. This book is very important because it expounds Ha-Levi's understanding of *Hilkhot Kiddush ha-Hodesh*, according to ancient astronomy and Ha-Levi's important achievements which would otherwise remain unknown. Most of Baneth's achievements are already gathered in his work. [Please note that Professor Eduard (Ezekiel) Baneth, 1855-1930, was a Talmudic scholar and Rabbi graduated from Hildesheimer Rabbinic Seminary, professor at the Lehranstalt fur die Wissenschaft des Judentum. He was the author of the monumental *Maimuni's Neumondsberechnung*.] Raphael Ha-Levi considered that the book was rather a textbook supporting oral teaching, but it was not ready for publication and could not be called a book. This book had nevertheless a considerable impact because the study of the chapters of *Kiddush ha-Hodesh* was still part of the curriculum of many Talmudic students. The Gaon of Vilna learned astronomy from this book (*Aliyot Eliyahu*, Levin Epstein 1954, p. 44). It served certainly, together with *Luhot ha-Ibbur*, as a reference book to the authors of subsequent books on the same subject:

- *Na'avah Kodesh* by Rabbi Simon Waltsch, Berlin 1786. The author belonged to the tradition of Rabbi Raphael Ha-Levi.
- *Kenei Middah* by Rabbi Barukh of Shklov, Prague 1784. The author belonged to the circle of the Gaon of Vilna but he studied medicine in Frankfurt-am-Oder and was certainly aware of Ha-Levi's books.

2. *Luhot ha-Ibbur* Vol. 1: Tables based on modern astronomy, Leiden, 5516.[6]
3. *Luhot ha-Ibbur* Vol. 2: Tables based on Maimonides' *Hilkhot Kiddush ha-odesh*, Hanover 5517. This book was printed a second time together with a commentary (definitions and explanations about using the tables and additional examples) in Vol. 2 about *Hilkhot Kiddush ha-Hodesh* by Meir Fürth in 1820–1821, under the name *Yirat Shamayim*, Dessau, 1820–1821.
4. Vorbericht vom Gebrauch der neuerfundenen logarithmische Wechsel-Tabellen....verfertiget und hrsg von Raphael Levi, Hannover, 1747.[7]
5. Raphael Levi: Rechnungsmethode Hrsg von Meyer Aaron Mit einer Abhandlung über die Vier Species des Rechnens mit Brüchen. Hannover, 1783.[8]
6. A table of the times of sunset and the stars' apparition throughout the year. Probably the first timetable constructed on an astronomical basis. Hanover, 1766.[9]
7. Calculus Differentialis oder Rechnung des Unendlichen des Herrrn von Leibnitz. Raphael Levi, Hanover 1776 (Library of the University of Hanover).

The following various unpublished works still remain in manuscript, scattered in different European libraries:

1. Zentralbibliothek Zürich: Ms Heid. 180. This manuscript corresponds to the printed book *Luhot ha-Ibbur* I. The manuscript is dated 1752 while the printed book is dated 1756.
2. Staatsbibliothek Berlin: Manuscript N° 255, 4d. This unpublished manuscript includes only two pages; the two faces of one leaf. Its title is:
חשבון הקץ והתחיה על ידי התוכן הפלסוף אלהי וטבעי מו"ה רפאל סגל מהנובר

- *Amudei Shamayim* by Rabbi Barukh of Shklov, Berlin 1777.
- *Yirat Shamayim* by Rabbi Meir Fürth מאיר פיורדא, Dessau, 1820. This book is a commentary on *Luhot ha-Ibbur*, Vol. 2.

It is surprising that so many books were published at the same period on the subject and that suddenly so many authors understood the subject. The book *Shevilei de-Raquiah* by Rabbi Eliyahu ha-Cohen Hechim, Prague 1784, belongs to the same period but its definition of the arc of vision is incorrect. He was still influenced by the Greek definition of the *arcus visionis*.

6 *Aliyot Eliyahu*, p. 47, tells that the Gaon of Vilna showed to the visiting author of *Homot Yerushalayim* a mistake that he found in a table of *Luchot ha-Ibbur*. This was sufficient to make his authority in this matter felt.

7 This book is registered in the Royal Library of Den Haag under the number 1116 B9.

8 This book is registered in the Royal Library of Amsterdam under the number Ros 1885 G 39.

9 See appendix at the end of the article.

It is a calculation of the time of the redemption based on Daniel. It was described by Steinschneider in Verzeichnis der Hebraïschen Handschriften... Berlin Königliche Bibliothek. Berlin 1878–1897.[10]

3. The Jews' College, London: Ms N° 134 according to the Catalogue of Hebrew manuscripts in the Jews' College. A. Neubauer, London, 1886. This manuscript corresponds to the book *Tekhunat ha-Shamayim*. It was written in 1734, and is identical to the printed book.
4. According to A. Neubauer: Catalogue of the Hebrew manuscripts in the Bodleian Library. Oxford, 1886–1908.
   - N° 2062: ספר תכונת השמים Includes 64 folios. This manuscript, probably autographic, begins with the text of the printed edition, but it is much longer and extended.
     Ox 2062 (Cat Neugebauer); Ox Mich 603; Ox Mich 847 (old n°).
   - N° 2063: חכמת התכונה Includes 45 folios. This unpublished manuscript deals with the principles of spherical astronomy.
     Ox 2063 (Cat Neugebauer); Ox Mich 498; Ox Mich 301 (old n°).
     Engineer Eran Raviv found a parallel manuscript in Moscow: MS Guenzburg 1743.
   - N° 2290:6: כללי סוד העיבור This unpublished manuscript deals with the rules of the calendar.
     Ox 2290 (Cat Neugebauer); Ox Mich 58; Ox Mich 345 (old n°).

The inscriptions on the graves of Rabbi Raphael Hanover and his wife provide much insight into their characters.

The inscription on Mrs. Hanover's grave is the following:[11]

פ"ט

האשה הצנועה והחסודה הגונה וספונה צדקת ה' עשתה בביתה ומעונה כפה פרשה לעני
בחסד וחנינה, לא פסקה פיה מתפילות ובקשות בכונה בכל עת ועונה פיה פתחה בחכמה ה"ה
מרת פאגיל בת התורני הרבני מוהר"ר ברוך זצ"ל והיא היתה אשת התורני כהר"ר רפאל סג"ל
שיחי' יצאה נשמתה ביום ב' ו' אלול תק"ל לפ"ק ת'נ'צ'ב'ה'

10 Apparently a translation in English. "The calculation of the end of the days" was issued in London in 1768. It fixed this year to 1783. In *Ma'amar Binah Le'itim* (London 1795), Elyakim ben Abraham – the Hebrew name of Jacob Hart (1745–1814) – based himself on the interpretation of Raphael Ha-Levi from Hanover (whom he did not credit) and connected this date to the Treatise of Versailles (1783) ending the war of America. The dream of Messianic redemption had begun in 1783 and would have its culmination in 1840.

11 Gronemann, Selig (1843–1918): Genealogishe Studien uber die alten judischen Familien Hannovers, Berlin 1913.

The inscription on Rabbi Raphael Hanover's grave is the following:[12]

פ"ט
איש אשר אלה לו ראוי להציב ציונים ולחוק
בעט ברזל למען ידעו דורות אחרונים איש צדיק
וישר ונשוא פנים בישישים חכמה ואורך
ימים ושנים, נהירין ליה שבילי דרקיע כשבילי
דנהר דעים ונבונים, יסיק שמים במרכבת תחכמונים
ואסף בחפניו כל גלילות ארץ וימים קדמונים חכמתו
ובינתו לעיני כל עמים והמונים, התיצב לפני מלכים
ורוזנים, ראוי לעבר את השנים רפאל אחד מן השרים
הראשונים ה"ה התורני הרבני המפורסם, מהור"ר רפאל
בן החבר רבי יעקב יוסף הלוי זצ"ל יצאה נשמתו ביום ב'[13]
לעת ערב ונקבר למחרתו ביום ג' ג' סיון תקל"ט לפ"ק
ת'נ'צ'ב'ה'

Picture of Rabbi Raphael Ha-Levi from Hanover

During this period it was common practice that people in Italy, as well as Germany, shaved (see Responsa Yabetz I: 80). Even famed Italian rabbis shaved: see the pictures of Rabbi

12 The inscription of his grave has been reconstructed from two deficient versions, the first in Gronemann, mentioned above and the second in S. E. Blogg's *Sefer ha-Hayim*. This last book contains prayers for sick persons and for deceased persons at the cemetery. At the end it mentions the inscription of the graves of some celebrated rabbis: Rabbi Meir of Rottenburg, Rabbi Jacob Emden, Rabbi Jonathan Eibeshutz, Rabbi Zelig Kara from Hanover, and Raphael Ha-Levi from Hanover. Finally, the text was corrected thanks to a picture of the tombstone found on the website http://www2.iag.uni-hannover.de/~kass/ by Eran Raviv.

13 May 17, 1779.

Samson Morpurgo (1681–1740), Rabbi Moses Gentily (1663–1711), and Rabbi Raphael Meldola (1754–1828). People in contact with gentile society were also obligated to wear wigs. Rabbi Menahem Azaria de Fano is also said to have shaved, while Rabbi Samson Morpurgo and Rabbi Raphael Meldola even wore wigs. Rabbi Samson Morpurgo was a celebrated rabbi, mentioned in *Shem ha-Gedolim* for his book of responsa called *Shemesh Tsedaka*. He was often consulted by Rabbi Isaac Lampronti in *Pachad Istshak*, and Rabbi Raphael Meldola, the Haham of London, had received rabbinical ordination from Rabbi H. J. D. Azulai.

Despite his appearance, which today could raise some contestation and interrogation, Raphael Ha-Levi was highly revered and respected by Jews and non-Jews alike. A rabbi as respected as Rabbi Beirush Bernstein (the grandson of Rabbi Joshua Falk (the Pnei Joshua) was proud to be Hanover's pupil, and the Gaon of Vilna studied astronomy in his books (*Aliyot Eliyahu*, pp. 44 and 47, and *Sefer ha-Gra* from R. Yehuda Leib ha-Cohen Maimon p. 33, two last lines).

The inscription in the Memorial book of the Jewish community of Hanover is as follows:[14]

יזכור אלקים את נשמת איש צדיק וישר ונשוא פנים בישישים חכמה ואורך ימים ושנים, נהירין ליה שבילי דרקיע כשבילי דנהר דעים ונבונים, יסיק שמים במרכבת תחכמונים ואסף בחפנו כל גלילות ארץ וימים קדמונים, חכמתו ובינתו לעיני כל עמים והמונים, יתיצב לפני מלכים ורוזנים, ראוי לעבר את השנים, רפאל אחד מן השרים הראשונים ה״ה התורני הרבני המפורסם, כל ימיו עסק במצות וגמילות חסדים מוהר״ר רפאל בן החבר רבי יעקב יוסף הלוי זצ״ל יצאה נשמתו ביום ב׳ לעת ערב ונקבר ביום ג׳ ג׳ סיון תקל״ט לפ״ק

Scant biographical elements of Rabbi Raphael Hanover's life are scattered through various books and journals.[15]

14 Gronemann, Selig: Genealogische Studien uber die alten judischen Familien Hannovers. Berlin, 1913.

15 1. Altmann, Alexander. *Moses Mendelssohn*, London 1973, pp. 161-163,786-788.
2. Blogg, S.E. *Sefer ha-Hayim*. Hanover 1848, p. 314. This very popular prayer book for sick persons, mourning and cemetery had 11 re-editions, the last one by Goldschmidt, Basel, 1983, but without the grave inscriptions.
3. Cohn, Berthold. *Jahrb. Der Juedische Literatuur Geschichte*. Vol. 18, 1927.
4. *Der Orient*, 7 n° 33, pp. 256-258.
5. Furst, Julius (1805–1873), *Bibliotheca Judaica*, Leipzig 1849–1863.
6. Gronemann, Selig. *Genealogishe Studien uber die alten judische Familien Hannovers*, Berlin, 1913
Erste Abteilung: *Genealogie der Familien*. Zweite abteilung: *Grabschriften und Gedächnisworte*.
7. Guhrauer, Gottschalk Eduard (1809–1854). Gottfried Wilhelm Freiherr v. Leibnitz. Breslau, 1846.

Raphael Hanover had the reputation of an extraordinary skilled calculator, of a rabbinical scholar, and a divine[16] and natural[17] philosopher. One can have an idea of the respect in which he was held and the high reputation he had by reading the rabbinical approbations to the books *Tekhunot ha-Shamayim*[18] by Rabbi Saül Loewenstamm (1717–1790)[19] and Rabbi Isaac Hayim Ibn Dana di Brito[20] from Amsterdam, as well as the approbations to the book *Na'ava Kodesh*[21] by Rabbi Tsvi Hirsch Levin[22] (1721–1800) from Berlin[23], Rabbi Arye Leib (1715–1789),[24] ben Jacob Joshua Falk (1680–1756),[25] and Rabbi Issachar Beirush Bernstein (1747–1802),[26] who was the latter's son, both being rabbis of Hanover. Similarly *Aliyot Eliyahu*, a book which is an ode to the glory of the Gaon of Vilna, tells that the Gaon learned astronomy in his book *Tekhunat ha-Shamayim*.[27] It also shows

8. *Literatuurblatt des Orient*, 1849, pp. 140-143.
9. Mensel Johan Georg, *Lexikon der von Jahr 1750–1800*. VIII, Leipzig, 1808.
10. Rohrbein, Waldemar R. *Judische Persönlichkeiten in Hannovers Geschichte*. Hannover, 1998.
11. Schulze, Peter. *Beitrage zur Geschichte der Juden in Hannover*. Hannover, 1998.
12. Steinschneider, Moritz. *Die Mathematik bei den Juden. Bibliotheca Mathematica.*
    N.F. Vol. 10 (1896) p. 38.
    N.S. Vol. 7-13 (1893–1899).
13. Steinschneider, Moritz. *Die Mathematik bei den Juden.*
    MGWJ 49 n° 13 (1905) pp. 723-728.
14. Zeitlin. *Bibl. Post Mendelssohn*, p. 135.
15. Zinberg. *Toledot Sifrut Yisrael*. Vol. 3, p. 366 and Vol. 5, p. 286.
16. Zuckerman, M. *Dokumente zur Geschichte der Juden in Hannover*. Hannover, 1908.
17. Schwarzschild, Steven and Henry Schwarzschild, "Two Lives in the Jewish Frühaufklärung: Raphael Levi Hannover and Moses Abraham Wolff", *Leo Baeck Year Book* 29 (1984), pp. 229-258.

16 A theologian.

17 A physicist and astronomer. Physics was called "natural" philosophy.

18 Amsterdam, 1756.

19 מגלת ספר שחיבר חכם גדול בחכמת התכונה אשר בזמננו ושמו כהר״ר רפאל הלוי מק״ק הנובר ונקרא בשם תכונת השמים

20 ה'ה התורני כהר״ר רפאל הלוי נר״ו מק״ק הנובר ה״י

21 Berlin, 1786.

22 The younger brother of Saül Loewenstamm, both sons of Rabbi Arié Leib Loewenstamm from Amsterdam (1690–1755), and nephews of Rabbi Jacob Emden (1697–1776).

23 החכם השלם התורני הרבני המנוח כמהו' רפאל הנובר זצ״ל אשר נודע ומפורסם גודל חכמתו בחכמה זו וכבר יצא מוניטין שלו בעולם כי הפליא לעשות קונטריסין אשר המה אוזנים לחכמה זו

24 ששמעתי נאמנה מפי החכם השלם המפורסם מוהר״ר רפאל סג״ל מכאן שהתפאר את בני הגאון שלדעתו בני ממש כיחיד בדורו בענין זה

25 The author of *Pnei Joshua*.

26 He learned *Hilkhot Kiddush ha-Hodesh* under Raphael ha-Levi Hanover.

27 R. Joshua Heshil ben Elijah Ze'ev ha-Levi Lewin (Vilna 1818–Paris 1883), *Aliyot*

the mathematical abilities of the Gaon of Vilna by the fact that he found a mistake in the book *Luhot ha-Ibbur*.[28]

Highlights of *Luhot ha-Ibbur*, Part I

HaLevi's book, *Luhot ha-Ibbur* consists of tables that were constructed according to the principles of modern astronomy, i.e. the astronomy of the beginning of the eighteenth century.

Glossary

מולד הנכון	The astronomical mean conjunction (corrected *molad*).
מסלול השמש	Sun's mean anomaly = sun's mean longitude minus apogee's longitude.
מסלול הירח	Moon's mean anomaly = moon's mean longitude minus apogee's longitude.
מסלול הרוחב	Moon's argument of latitude F = longitude of the moon minus longitude of the ascending node. In our tables Hanover tabulates 2F.
מנת מסלול השמש	Sun's quota of the anomaly = equation of the center.
מנת מסלול הירח	Moon's quota of the anomaly = equation of the center.
מנת הזמן	Angular velocity expressed in "/hour.
מסלול הירח המתוקן	Corrected moon's argument of latitude.
איכות	Parity of the argument of latitude or of its variation: Even means that $F > 2k*180°$: the moon's latitude is positive. Uneven: $F > (2k+1)*180°$: the moon's latitude is negative.
תקופה אמיתית	True equinox.
תקופה נכונה	Mean equinox (different from *tekufa* of Samuel and Adda).

Eliyahu, Vilna 1856, p. 44. In fact the information was copied nearly verbatim from the introduction by the Gra's sons Avraham and Yehuda Leib to the book *Aderet Eliyahu*, Dubrovna, 1804. See also: R. Yehuda Leib ha-Cohen Maimon, *Sefer ha-Gra*, Jerusalem 1971, p. 33, last lines and Eliyahu Stern, *The Genius, Elijah of Vilna and the Making of Modern Judaism*, Yale Judaica Press, 2013, pp. 11, 37-39, 44 and 194. Note that early manuscripts of the future *Tekhunat ha-Shamayim* circulated already as early as 1727. This justifies that R. Elijah, born in 1720, could have known this book, still in manuscript, at a very young age. (Thank you Eran Raviv for this precision.)

28 *Aliyot Eliyahu*, p. 47. I have always asked myself what was the mistake discovered? Was it a misprint, an arithmetical mistake, or an astronomical mistake? (See also the commentary on tables 6 and 7.)

Definitions

L: Geocentric mean longitude of the sun.
L': Geocentric mean longitude of the moon.
Ω: Mean longitude of the moon's ascending node.
Γ: Longitude of the sun's perigee.
Γ': Longitude of the moon's perigee.
$D = L - L'$: Moon's mean elongation.
$M = L - \Gamma - 180°$: Sun's mean anomaly. Today $M = L - \Gamma$.
$M' = L' - \Gamma' - 180°$: Moon's mean anomaly. Today $M' = L' - \Gamma'$.

Astronomical References

The following references should be consulted in order to better understand HaLevi's book, *Luhot ha-Ibbur*:

1. The Equation of Time in Ancient Jewish Astronomy: J. J. Ajdler, *B.D.D.* 16, pp. 43-51.
2. Syzygies Tables: Jean Meeus, Kessel-Lo, 1963.[29]
3. Textbook on Spherical Astronomy: W. M. Smart. Cambridge University Press. This book was reedited many times.

29 This book was decisive for understanding the signification of Hanover's tables.

Reprinted Tables from Luhot ha-Ibbur

א למחזורים

	מולדות (יום. ש. חל)	תיקומים (שעות. חלק)	יתרונות (יום. ש. חל)	מסלול השמש (מספרים)	מסלול הירח (מספרים)	מסלול הרוחב (מספרים)	מסלול הרוחב (איכות)
תיקור	2. 5. 204	. 3411.	15. 2. 235	5801	9223	4779	1
א	2. 16. 595	0. 10. 46	0. 2. 40	12951	11050	545	2
ב	5. 9. 110	0. 21. 31	0. 4. 80	12943	9139	1091	2
ג	1. 1. 705	0. 32. 17	0. 6. 120	12934	7229	1636	2
ד	3. 18. 220	0. 43. 2	0. 8. 160	12925	5319	2181	2
ה	6. 10. 815	0. 53. 48	0. 10. 200	12917	3408	2727	2
ו	2. 3. 330	0. 64. 33	0. 12. 240	12908	1498	3273	2
ז	4. 19. 925	0. 75. 19	0. 14. 280	12899	12548	3818	2
ח	7. 12. 440	0. 86. 4	0. 16. 320	12890	10637	4364	2
ט	3. 4. 1035	0. 96. 50	0. 18. 360	12882	8727	4909	2
י	5. 21. 550	0. 107. 35	0. 20. 399	12873	6817	5454	2
כ	4. 19. 20	0. 215. 11	1. 16. 799	12786	673	10909	2
ל	3. 16. 570	0. 322. 46	2. 13. 118	12699	7490	3403	1
מ	2. 14. 40	0. 430. 22	3. 9. 518	12612	1347	8858	1
נ	1. 11. 590	0. 537. 57	4. 5. 917	12525	8163	1352	2
ס	7. 9. 60	0. 645. 52	5. 2. 236	12438	2020	6807	2
ע	6. 6. 610	0. 753. 8	5. 22. 636	12351	8837	12261	2
פ	5. 4. 80	0. 860. 43	6. 18. 1035	12264	2694	4756	1
צ	4. 1. 630	0. 968. 19	7. 15. 355	12177	9510	10210	1
ק	2. 23. 100	0. 1075. 54	8. 11. 754	12090	3367	2704	2
ר	5. 22. 200	1. 1071. 48	16. 23. 428	11221	6734	5409	2
ש	1. 21. 300	2. 1067. 42	25. 11. 102	10351	10101	8113	2
ת	4. 20. 400	3. 1063. 36	33. 22. 856	9481	508	10818	2

כם בחשבון היתרונות כשעברו ח שנים מן המחזור צריך להוסיף על יתרונות מן המחזורים 1. 14. 172 דהיינו יום אחד יד׳ שעות קעב חלקים

ד לוח מסלול השמש

	לימים	לשעות
א	35	1
ב	71	3
ג	106	4
ד	142	6
ה	177	7
ו	213	9
ז	248	10
ח	284	12
ט	319	13
י	355	15
יא	390	16
יב	426	18
יג	461	19
יד	497	21
טו	532	22
טז	568	24
יז	603	25
יח	639	27
יט	674	28
כ	710	30
כא	745	31
כב	781	33
כג	816	34
כד	852	55
כה	887	
כו	923	
כז	958	
כח	993	
כט	1029	

Table 1: Mean Movements of the Sun and Moon, the *Molad*, the Corrections and Supplements during 19-Year Cycles.
Table 4: Mean Movement of the Sun's Anomaly during Hours and Days.

ה לוח מסלול הרוחב

מסלול	חלקים	מסלול	שעות
1	36	40	א
3	72	79	ב
4	108	119	ג
5	144	159	ד
7	180	198	ה
8	216	238	ו
9	252	278	ז
11	288	318	ח
12	324	357	ט
13	360	397	יוד
15	396	437	יא
16	432	476	יב
17	468	516	יג
19	504	556	יד
20	540		
21	576		
22	612		
24	648		
25	684		
26	720		
28	756		
29	792		
30	828		
32	864		
33	900		
34	936		
36	972		
37	1008		
38	1044		

ב לשנים

מסלול הרוחב		מסלול הירח	מסלול השמש	יתרונות	תיקונים	מילדות	
איכות	מספרים	מספרים	מספרים	חלקים ש' יו'	שעו' חל'	הל' ש' ימים	
2	579	11153	12574	10. 21. 6	0. 33	4. 8. 876	א
2	1159	9346	12187	21. 18. 12	1. 6	1. 17. 672	ב
2	3946	8468	12848	3. 2. 305	1. 42	7. 15. 181	ג
2	4526	6661	12462	13 23. 310	2. 15	4. 23. 1057	ד
2	5105	4854	12075	24. 20. 316	2. 48	2. 8. 853	ה
2	7893	3976	12737	6. 4. 628	3. 23	1. 6. 362	ו
2	8472	2169	12350	17. 1. 620	3. 56	5. 15. 158	ז
2	11260	1291	51	עיין במחזורים בסימן	4. 32	4. 12. 747	ח
2	11839	12444	12625	9. 6. 914	5. 5	1. 21. 543	ט
0	12418	10637	12239	20. 3. 919	5. 38	6. 6. 339	יוד
1	2246	9759	12900	1. 12. 132	6. 14	5. 3. 928	יא
1	2825	7952	12513	12. 9. 138	6. 47	2. 12. 724	יב
1	3405	6144	12127	23. 6. 144	7. 20	6. 21. 520	יג
1	6192	5267	12788	4. 14. 437	7. 55	5. 19. 29	יד
1	6792	3460	12402	15. 11. 443	8. 28	3. 3. 605	טו
1	7351	1652	12015	26. 8. 448	9. 1	7. 12. 701	טז
1	10138	775	12676	7. 16. 741	9. 37	6. 10. 210	יז
1	10718	11027	12290	18. 13. 747	10. 10	3. 19. 6	יח

ג לחדשים

מסלול הרוחב		מסלול הירח	מסלול השמש	יתרונות	תיקונים	מילדות	
2	2209	929	1048	0. 21. 810	0. 3	1. 12. 793	א
2	4417	1859	2096	1. 19. 541	0. 5	3. 1. 506	ב
2	6625	2788	3143	2. 17. 271	0. 8	4. 14. 219	ג
2	8833	3718	4191	3. 15. 2	0. 11	6. 2. 1012	ד
2	11041	4647	5239	4. 12. 812	0. 14	7. 15. 725	ה
1	290	5576	6287	5. 10. 543	0. 16	2. 4. 438	ו
1	2498	6506	7335	6. 8. 273	0. 19	3. 17. 151	ז
1	4706	7435	8382	7. 6. 4	0. 22	5. 5. 944	ח
1	6915	8365	9430	8. 3. 814	0. 25	6. 18. 657	ט
1	9123	9294	10478	9. 1. 545	0. 27	1. 7. 370	יוד
1	11331	10223	11526	9. 23. 275	0. 30	2. 20. 83	יא
2	579	11153	12574	10. 21. 6	0. 33	4. 8. 876	יב
1	1104	6945	524			0. 18. 396	חצי חודש

Table 2: Mean Movements of the Sun and the Moon, the Molad, the Corrections and Supplements during Years of the Cycle.
Table 3: Mean Movements of the Sun and Moon, the Molad, the Corrections and Supplements during the Months.
Table 5: Variation of the Argument of the Moon's Latitude during Hours and Halakim. Hanover tabulates 2F, i.e. twice the moon's argument of latitude.

לוח מנת המסלול ומנת הזמן לשמש													
	0		1080		2160		3240		4320		5400		
לגרות	מנת המסלול	מנת הזמן	מנת המסלול	מנת הזמן	מנת המסלול	מנת הזמן	מנת המסלול	מנת הזמן	מנת המסלול	מנת הזמן	מנת המסלול	מנת הזמן	
0	0	143	3641	143	6350	146	7425	148	6503	151	3784	153	1080
36	172		3751		6415		7426		6439		3668		1044
72	254		3858		6488		7424		6373		3553		1008
108	331		3967		6549		7423		6305		3437		972
144	506		4074		6609		7418		6235		3320		936
180	633	143	4183	144	6666	146	7407	148	6163	152	3200	153	900
216	758		4283		6721		7399		6087		3079		864
252	883		4385		6775		7386		6012		2960		828
288	1010		4487		6826		7374		5933		2839		792
324	1137		4589		6884		7357		5853		2716		756
360	1262	143	4689	144	6926	146	7337	149	5770	152	2592	153	720
396	1388		4789		6968		7319		5691		2468		684
432	1512		4884		7011		7296		5606		2342		648
468	1637		4980		7053		7269		5519		2216		612
504	1761		5075		7093		7241		5430		2090		576
540	1885	143	5167	144	7130	147	7211	150	5338	152	1962	153	540
576	2008		5257		7165		7181		5245		1834		504
612	2126		5346		7197		7146		5151		1705		468
648	2245		5434		7228		7109		5055		1577		432
684	2369		5521		7256		7070		4958		1448		396
720	2490	143	5605	145	7281	147	7029	150	4859	152	1318	153	360
756	2608		5690		7305		6987		4700		1187		324
792	2727		5771		7329		6942		4657		1058		288
828	2838		5852		7349		6895		4553		926		252
864	2959		5929		7368		6815		4447		794		216
900	3075	143	6003	145	7382	148	6792	151	4340	153	662	153	180
936	3190		6074		7395		6739		4232		529		144
972	3304		6144		7406		6683		4121		398		108
1008	3421		6215		7415		6624		4010		265		72
1044	3526		6254		7421		6563		3897		133		36
1080	3641	143	6350	146	7425	148	6503	151	3784	153	0	153	0
	11880.		10800.		9720.		8640.		7560.		6480.		להוסיף

Table 6: The Sun's Quota of the Anomaly (in units of 100 Seconds of Arc) or the Equation of the Centre and its Angular Velocity (in Seconds of Arc per Hour) as a Function of the Sun's Anomaly.
The sun's angular velocity is a function of the anomaly expressed in units of 100″ = 0.027778°. Thus 360 = 10°, 720 = 20°, 1080 = 30°, 6480 = 180°. The quota is negative when the anomaly < 180° and positive when the anomaly > 180°. The anomaly is measured from the apogee; anomaly = M + 180°. For the anomaly 1°: read 127 instead of 172. For 59°: read 6284 instead of 6254. For 114° read 6845 instead of 6815.

לוח מנת המסלול ומנת הזמן לירח

	0		1080		2160		3240		4320		5400		
לגרעין	מנת המסלול	מנת הזמן	מנת המסלול	מנת הזמן	מנת המסלול	מנת הזמן	מנת המסלול	מנת הזמן	מנת המסלול	מנת הזמן	מנת המסלול	מנת הזמן	
0	0	1814	8588	1832	15160	1891	17970	1966	16009	2058	9431	2131	1080
36	298	1814	8852	1834	15317	1893	17988	1969	15859	2061	9148	2132	1044
72	596	1814	9110	1836	15476	1895	17990	1972	15703	2064	8862	2134	1008
108	893	1814	9367	1837	15628	1897	17999	1975	15541	2067	8574	2136	972
144	1190	1815	9622	1839	15780	1898	17998	1979	15374	2069	8283	2137	936
180	1488	1815	9875	1840	15944	1900	17989	1982	15202	2072	7989	2138	900
216	1784	1815	10128	1841	16064	1902	17976	1985	15025	2075	7693	2139	864
252	2079	1815	10373	1843	16200	1905	17960	1989	14844	2078	7394	2140	828
288	2374	1816	10614	1844	16332	1907	17931	1993	14658	2080	7092	2142	792
324	2670	1816	10858	1846	16459	1910	17904	1996	14468	2083	6787	2143	756
360	2965	1817	11098	1848	16581	1913	17868	2000	14273	2085	6480	2144	720
396	3259	1817	11333	1850	16698	1915	17826	2004	14072	2088	6171	2146	684
432	3553	1818	11566	1851	16812	1918	17786	2008	13867	2090	5860	2147	648
468	3844	1818	11795	1853	16918	1921	17727	2010	13658	2093	5547	2148	612
504	4136	1819	12022	1855	17021	1923	17670	2013	13444	2095	5231	2149	576
540	4425	1819	12245	1857	17119	1926	17607	2015	13227	2098	4914	2150	540
576	4713	1820	12464	1858	17213	1929	17539	2017	13003	2100	4595	2150	504
612	5000	1820	12680	1860	17302	1931	17463	2020	12775	2103	4274	2151	468
648	5286	1821	12893	1862	17384	1934	17385	2022	12542	2105	3952	2152	432
684	5570	1822	13102	1864	17462	1937	17301	2025	12305	2107	3628	2152	396
720	5852	1823	13307	1866	17535	1939	17211	2028	12065	2110	3302	2153	360
756	6133	1824	13509	1868	17602	1942	17116	2031	11819	2113	2974	2153	324
792	6413	1824	13708	1870	17663	1945	17014	2034	11568	2115	2646	2154	288
828	6693	1825	13903	1872	17720	1947	16907	2037	11314	2117	2317	2154	252
864	6969	1826	14095	1874	17772	1950	16796	2040	11056	2119	1988	2155	216
900	7245	1827	14283	1877	17819	1953	16678	2043	10793	2121	1658	2155	180
936	7518	1828	14466	1880	17858	1955	16556	2046	10527	2123	1328	2155	144
972	7789	1829	14645	1883	17894	1958	16427	2049	10258	2125	997	2156	108
1008	8058	1830	14820	1885	17925	1961	16293	2052	9985	2127	664	2156	72
1044	8324	1831	14991	1888	17950	1963	16153	2055	9710	2129	332	2156	36
1080	8588	1832	15160	1891	17970	1966	16009	2058	9431	2131	0	2157	0
	11880.		10800.		9720.		8640.		7560.		6480.		לחוסף

Table 7: The Moon's Quota of the Anomaly (in units of 100 Seconds of Arc) at the Conjunction or Opposition, or the Equation of the Centre Diminished by the Evection, and the Moon's Angular Velocity (in Seconds of Arc per Hour) as a Function of the Moon's Anomaly Expressed in Units of 100″ = 0.02778°.
Thus 360 = 10°, 720 = 20°, 1080 = 30°, 6480 = 180°. The quota is negative for anomaly < 180° and positive for anomaly > 180°. The anomaly is measured from the apogee; anomaly = M' + 180°.

ח

לוח מנת ורוחב המדינה

מז מורה מזרחית / מע מורה מערבית } לירושלים { ומנת המדינה להוסיף / ומנת המדינה לגרוע

שמות המקומות			מנת המדינה שעות	מנת המדינה חלקים	רוחב צפונית מעלות	רוחב צפונית חלקים
אלעכסנדריא	מצרים	מע	.0.	364	31.	11
אמשטרדאם	הולאנדיא	מע	.2.	125	52.	23
ברלין	אשכנז	מע	.1.	795	52.	33
ברעסלויא	אשכנז	מע	.1.	530	51.	3
גאא	הודו מזרחית	מז	.2.	606	15.	31
דאנציג	פאלוניא	מע	.1.	431	54.	22
האמבורג	אשכנז	מע	.2.	28	53.	41
האנאבר	אשכנז	מע	.2.	85	52.	27
ווארשויא	פאלוניא	מע	.0.	1026	52.	14
ווינא	אשכנז	מע	.1.	493	48.	13
ווענעדיג	איטליא	מע	.1.	595	45.	25
טורין	איטליא	מע	.1.	912	44.	50
ירושלים					31.	50
לונדאן	אנגליא	מע	.2.	390	51.	31
לייפציג	אשכנז	מע	.1.	840	51.	19
לימא	הודו מערבית	מע	.7.	515	ד׳.12.	1
ליסבאן	פורטוגאליא	מע	.2.	1041	38.	42
מאדריט	ספרד	מע	.2.	653	40.	25
מארנא	איטליא	מע	.1.	637	44.	34
מאסקויא	יון	מז	.0.	360	55.	36
מעטץ	לאטרינגן	מע	.2.	319	49.	7
מענץ	אשכנז	מע	.2.	124	49.	54
נאפאלים	איטליא	מע	.1.	408	40.	51
נירן בערג	אשכנז	מע	.1.	927	49.	26
סיאם	הודו מזרחית	מז	.4.	396	14.	18
עדינבורג	שאטלאנדיא	מע	.2.	906	55.	58
פאדוא	איטאליא	מע	.1.	905	45.	22
פאריז	צרפת	מע	.2.	516	48.	50
פעטרס בורג	יון	מע	.0.	660	60.	0
פלארענץ	איטאליא	מע	.1.	589	43.	47
פעקין	הודו מזרחית	מז	.5.	435	39.	54
פראג	אשכנז	מע	.1.	811	50.	4
פראנקפורט	אשכנז	מע	.2.	66	49.	55
קאנדיא	איי איטאליא	מע	.0.	722	35.	19
קאנשטאנטינאפל	טירקא	מע	.0.	464	41.	0
קאפן האגן	דאניא	מע	.1.	840	55.	41
קראקויא	פאלוניא	מע	.1.	36	50.	10
ראמה	איטאליא	מע	.1.	866	41.	54
שטאקהאלם	שוועדן	מע	.1.	366	59.	20

גבולי הליקות

אם יהיה הגבול קרוב למספר השלם בין מלפניו ובין מלאחריו { בירח { פחות מן 864 אפשר שנלקה / פחות מן 768 ודאי נחשך / פחות מן 380 ודאי נחשך בכל שטחו } בשמש { פחות מן 1150 ודאי נלקה / פחות מן 1247 אפשר שנלקה }

ואם יהיה האיכות מספר זוג אזי הירח לצפון השמש { ונלקה השמש בחלקה העליון ועל הרוב נראית במדינת צפונית אם היא ביום ונחשך הירח בחלקו התחתון }

ואם יהיה האיכות מספר נפרד אזי הירח לדרום השמש { ונלקה השמש בחלקה התחתון ועל הרוב אינה נראית במדינת צפונית אפילו ביום ונחשך הירח בחלקו העליון }

ותמיד יהיה התחלת { ליקוי השמש בחלקה המערבי / ליקוי הירח בחלקו המזרחי }

מספר 12960 בין במסלול המאורות ובין במסלול הרוחב נקרא מספר השלם.

Table 8: Geographical Coordinates of the World's Major Cities, and Conditions for Solar and Lunar Eclipses.
The longitude is given in time east or west of Jerusalem, and the latitude is given in degrees. The table also gives rules for both solar and lunar eclipses.

א" מולדות	
עיקר	2. 5. 201
ד' מחזורים	5. 22. 200
פ' מחזורים	5. 4. 80
ס' מחזורים	3. 4. 1035
ה' שנים	2. 8. 853
ששה חדשים	2. 4. 438
מולד אמצעי	7. 1. 650

ב תיקונים	
ק' מחזורים	—. —. 1075. 54
פ' מחזורים	96. 50
י"ד שנים	7. 55
ששה חדשים	16
מנת התיקון	—. 1. 100. 55
מולד אמצעי הי"ת	7. 1. 650.
תגרע מנת התיקון	—. 1. 101.
מולד הנכון	7. 0. 549.

ג" מסלול המאורות לרגע המולד	לשמש ‖ לירח
עיקר	9223 ‖ 5801
ד' מחזורים	6734 ‖ 11221
פ' מחזורים	2694 ‖ 12264
ס' מחזורים	8727 ‖ 12882
ה' שנים	4854 ‖ 12075
ששה חדשים	5576 ‖ 6287
	37808 ‖ 60530
תגרע מספר השלם	25920 ‖ 51840
מסלול המאורות	11888 ‖ 6890
מנת מסלול הירח	8529 להוסיף
מנת מסלול השמש	6587 להוסיף
מרחק המאורות	1942 לגרוע
מסלול הירח הי"ת	11888
תגרע חצי המרחק	10
מסלול הירח המתוקן	11878
מנת הזמן לירח	1832
מנת הזמן לשמש	151
מרחק הזמן	1681
ויהיה הבדל המולד לגרוע	—. 1. 168
מולד הנכון הי"ת	7. 0. 549
תגרע הבדל המולד	—. 1. 168
מולד אמתי בירושלים	6. 23. 381
תגרע מנת המדינה	2. 85
מולד אמתי בהנובר	6. 21. 296

ד" מסלול המאורות לרגע הניגוד	לשמש ‖ לירח
מסלול המאורות במולד הי"ת	11888 ‖ 8690
תוסיף מסלול לחצי חודש	6945 ‖ 524
	18853
תגרע מספר השלם	12960
מסלול המאורות בניגוד	5873 ‖ 9214
מנת מסלול הירח	5503 לגרוע
מנת מסלול השמש	7239 להוסיף
מרחק המאורות	12742 להוסיף
מסלול הירח הי"ת	5873
תוסיף חצי המרחק	64
מסלול הירח המתוקן	5937
מנת הזמן לירח	2150
מנת הזמן לשמש	150
מרחק הזמן	2000
ויהיה הבדל הניגוד להוסיף	—. 6. 401
מולד הנכון הי"ת	7. 0. 549
תוסיף שארית חצי חודש	—. 18. 396
ניגוד הנכון	7. 18. 915
תוסיף הבדל הניגוד	—. 6. 401
ניגוד אמתי בירושלים	1. 1. 266
תגרע מנת המדינה	2. 85
ניגוד אמתי בהנובר	7. 23. 181

ה' מסלול הרוחב לשמש	
עיקר	1 ‖ 4779
ד' מחזורים	2 ‖ 5409
פ' מחזורים	1 ‖ 4756
ס' מחזורים	2 ‖ 4909
ה שנים	2 ‖ 5105
ששה חדשים	1 ‖ 290
חיבור המסלול	9 ‖ 25248
תוסיף כפל מנת הירח במולד	171
	25419
תגרע מסלול הרוחב לחבדל המולד	46
מסלול המתוקן	25373
תגרע מספר השלם	1 ‖ 12960
נבול לשמש	10 ‖ 12413

השמש נלקית ברגע מולד אמתי והואיל שהאיבורה מספר זוג יהיה הירח לצפון השמש ונלקית השמש בחלקה העליון ונראית

ד' מסלול רוחב הירח	
חיבור המסלול במולד הי"ת	9 ‖ 25248
תוסיף מסלול לחצי חודש	1 ‖ 1104
חיבור המסלול בניגוד	10 ‖ 26352
תגרע כפל מנת הירח לניגוד	110
	26242
תוסיף מסלול לחבדל הניגוד	253
מסלול המתוקן	26495
תגרע מספר השלם	2 ‖ 25920
נבול לירח	12 ‖ 575

הירח נלקה ברגע ניגוד האמתי. והואיל שמספר האיבורה זוג. יהיה הירח לצפון דרך השמש. ונחשך הירח בחלקו התחתון ונראית...

ו' יתרונות	
עיקר	15. 2. 235
ד' מחזורים	16. 23. 428
פ' מחזורים	6. 18. 1035
ס' מחזורים	—. 18. 360
ה' שנים	24. 20. 316
ששה חדשים	5. 10. 543
חיבור הראשון	45. 9. 14
תגרע חיבור השני	24. 12. 743
יתרון לחודש אדר שני	20. 20. 351
תוסיף מולד הנכון	7. 0. 549
תקופה נכונה י"ט אדר שני	6. 20. 900
מסלול השמש במולד הי"ת	8690
תוסיף המסלול לעשרים יום	710
לעשרים שעות	30
מסלול השמש לרגע התקופה	9430
מנת מסלול השמש	7373 להוסיף
מנת הזמן לשמש	149
ויהיה הבדל התקופה לגרוע	2. 1. 522
תקופה נכונה הי"ת	6. 20. 900
תגרע הבדל התקופה	2. 1. 522
תקופה אמתי בירושלים	4. 19. 378
תגרע מנת המדינה	2. 85
תקופה אמתי בהנובר י"ו אדר שני	4. 17. 293

והיינו תשצ"ג חלקי תתר"ף קודם חצות יום רביעי י"ו אדר שני תבא השמש לראש מזל טלה

Table 9: Calculations of the Situation at the Conjunction of March 1, 1737: Was It a Solar Eclipse? And the Situation at the Opposition of March 16, 1737: Was It a Lunar Eclipse? Calculation of the True Equinox of March 20, 1737.
In the right column, there is a misprint: the moon's anomaly is 8690 instead of 6890. We note that the solar anomaly is 11888 and its quota is + 6587, the lunar anomaly is 8690, and its quota is + 8529.

Description of Columns in Tables 1–3

Following is a description of the columns and rows in the following tables:

Table 1: Mean Movements of the Sun and Moon, the *Molad*, the Corrections and Supplements during 19-Year Cycles

Table 2: Mean Movements of the Sun and Moon, the *Molad*, the Corrections and Supplements during Years of the Cycle

Table 3: Mean Movements of the Sun and Moon, the *Molad*, the Corrections and Supplements during the Months

1st column: Number of cycles.

2nd column: *Molad* – Residue corresponding to the span of time defined in the first column for the calculation of the *molad.*

3rd column: Correction for the astronomical mean conjunction corresponding to the span of time defined in the first column. The mean astronomical conjunction, according to modern astronomy (in the beginning of the eighteenth century), does not perfectly coincide with the *molad* because the synodic mean lunar month is slightly shorter than the Jewish month of 29d 12h 793p. Therefore the mean conjunction occurs before the *molad.*[30]

4th column: Supplements representing the excess of the Jewish cycles of 19 years or 235 lunations on the tropical years during the span of time defined in the first column in order to calculate the exact length of the tropical years during the span of time defined in the first column.[31]

5th column: The variation of the sun's mean anomaly, i.e. the longitude of the mean sun minus the longitude of the sun's apogee,[32] during the span of time defined in the first column.

6th column: Variation of the moon's mean anomaly, i.e. the longitude of the mean moon minus the longitude of the moon's apogee, during the span of time defined in the first column.

7th column: Variation of 2F, i.e. twice the moon's argument of latitude during the span of time defined in the first column. F represents the distance between the

30 Before year 3411 AMI, the mean conjunction occurred after the *molad.* At the beginning of the Jewish calendar at the *molad* of *Beharad*, the mean conjunction had a delay of 1h 47.5m with regard to *Beharad.*

31 In order to calculate a mean equinox or a solstice.

32 In ancient astronomy and still in the eighteenth century, the anomaly is considered with regard to the apogee. In modern astronomy we refer to the perigee.

moon and the ascending node.
8th column: Parity of the variation of the argument of latitude. If the parity is even, then the moon's latitude beholds its sign and the moon remains on the same side with regard to the ecliptic. If the parity is uneven, then the moon's latitude changes its sign and the moon is now on the other side with regard to the ecliptic. If the parity is even, the moon's latitude is positive, and if it is uneven, then the moon's latitude is negative.
1st row: Gives the radices, or the different parameters at the epoch, i.e., the astronomical mean conjunction corresponding to the *molad* of *Beharad*. The addition of the radix of each parameter with the value of the variation of this parameter during the span of time corresponding to a certain line of the first column gives the value of this parameter after the end of this span of time counted from the astronomical mean conjunction corresponding to *Beharad*. The radices are the different values of the parameters at the moment of the astronomical conjunction corresponding to the *molad* of *Beharad*. The values of the radices were calculated by Hanover in such a way that the mean parameters calculated for his epoch correspond with the accepted astronomical values.

Justification for the Various Tables In *Luhot Ha-Ibbur*

This section provides justification for the various tables that Ha-Levi of Hanover has calculated in *Luhot Ha-Ibbur*.

Table 1. Mean Movements of the Sun and Moon, the Molad, the Corrections and Supplements during 19-Year Cycles

1. *Tikkunim* or corrections that allow one to find the astronomical mean conjunction distinct from the *molad*.

   For a span of 400 cycles of 19 years each, corresponding to 94,000 months, Hanover gives a correction of 3h 1063hal and 36/60 or 4,303.6 hal.[33] The correction for one lunation is then 0.045 782 978 723 4 hal or 0.152 609 92 s. The lunation of Hanover is thus, instead of 29-12-793, 29d 12h 792.954 217 021 277 hal or 29.530592369483 days. In other words, Hanover considers an astronomical month to be 29d 12h 44m 3.1807233s instead of the Jewish month of 29d 12h 44m 3.333s.
   This value is slightly higher than the following values mentioned by Lalande

33 1 *helek* = 3 1/3 seconds.

(1732–1807) in his Astronomy book published in 1764:
Ismael Bouillaud (1605–1694): 29d 12h 44m 3.1603s
Tobias Mayer (1723–1762): 29d 12h 44m 2.8897s
Hanover considers that the astronomical conjunction coincided with the *molad* in Tishri 3411. Therefore in Tishri 5516 at the date of the publication of his book, after 2105 years[34] or 26035 months after the epoch of coincidence,[35] the difference amounts to 26035 * 0.15260992 = 3973,19927s = 66.22m. The astronomical mean conjunction precedes the *molad* by 66.22m.

2. *Yitronot* or excesses represent the excess of the astronomical lunar years or cycles with regard to the tropical years.

Hanover gives for 400 cycles or 94000 lunations 33d 22h 856 hal.

94,000 Jewish months represent:	2,775,875.848	765	440	000	d
Correction for astronomical lunations, of 4303.6 hal	− 0.166	033	950	617	d
Length of 94000 astronomical lunations:	2,775,875.682	731	489	383	d
Excess on 7600 tropical years:	− 33.949	691	358	025	d
Length of 7600 tropical years:	2,775,841.733	040	131	358	d
Length of 19 tropical years:	6,939.604	332	605		d
Length of a tropical year:	365.242	333	295		d

If we compare the tropical year of Hanover with other historical data, we have the following elements:

Rabbi Adda	365d 5h 55m 25.4386s
Ptolemy, second century	365d 5h 55m 12s
Al-Battani, ninth century	365d 5h 46m 24s
Rabbi Abraham bar Hiyya (12th century)	365d 5h 55m 12s
Alphonsine Tables, 126	365d 5h 49m 16s
Copernicus (1473–1543)	365d 5h 49m 20s
Flamsteed (1646–1719)	365d 5h 48m 57.5s
Jacques Cassini (1677–1756)	365d 5h 48m 49s
De la Caille (1713–1762)	365d 5h 48m 49s
Lalande, 1764	365d 5h 48m 45s

34 2105 = 110*19 + 15. It corresponds to 110*235 + 10 * 12 + 5 * 13 = 26035 months.

35 This simplified calculation, which does not take into consideration the real leap years, gives a result which does not differ from the true number of elapsed months by more than one month. The consequence is insignificant.

Hanover, 1756	365d 5h 48m 57.6s
Tropical year 190	365d 5h 48m 45.97s

The tropical year of Hanover corresponds practically with the value of Flamsteed.

3. The sun's anomaly

   Hanover gives a value of 9481 for 400 cycles; it represents the variation of the sun's anomaly during 400 cycles or 94000 astronomical mean lunations.
   According to the data given by Jean Meeus,[36] the variation of the sun's mean anomaly in 36525 days is today 35,999°. 050 30. We know that 94000 Jewish months represent 2,775,875.848 765 43d and 94000 astronomical lunations represent, according to Hanover, 2,775,875.68273148d or 36525d * 75.999 334 229 4. During this last period the sun's anomaly increases by 7599 *360° + 263°.8557. If we transform this remainder in seconds of angle we obtain 949880", and dividing this by 100 results in 9499. This figure is very near to 9481 given by Hanover and it confirms the procedure of calculation. In fact, Hanover considers a variation of 35999°.043 792 3 in 36525 days, slightly different from the value of Meeus.

4. The moon's anomaly

   Hanover gives a value of 508 for 400 cycles. It represents the variation of the moon's anomaly during 94000 astronomical mean lunations.
   According to the data given by Jean Meeus,[37] the variation of the moon's mean anomaly in 36525 days is 477,198°.867 631 3. Now 94000 astronomical lunations represent, according to Hanover, 2,775,875.682 731 48d and correspond to 36525d * 75.999 334 229 4. During this period the moon's anomaly increases by 100741 * 360° +36.2350°. If we transform this remainder in seconds of angle we obtain 130446" and dividing this by 100 results in 1304.46.
   Again this figure is close to 508 given by Hanover. In fact, Hanover considers a variation of the moon's anomaly of 477,198°.576 525 in 36525 days which is very close to the modern value of Meeus.

36 Jean Meeus, Astronomical Algorithms, Willmann-Bell, chapter 24, p. 151.
37 Jean Meeus, Astronomical Algorithms, Willmann-Bell, chapter 45, p. 308.

5. The moon's argument of latitude

   Hanover gives a value of 10818 for 400 cycles, or 94000 astronomical mean lunations, and an even parity for the variation of the argument of latitude. As we'll see, Hanover tabulates twice the argument of latitude in his tables.

   According to the data given by Jean Meeus,[38] the variation of the moon's argument of latitude in 36525 days is 483,202°.017 527 3. Now 94000 astronomical lunations represent, according to Hanover, 2,775,875.682 731 48d and correspond to 36525d * 75.999 334 229 4. During this period the moon's argument of latitude increases by 102008 * 360° + 151°.6304. If we transform this remainder in seconds of angle we get 545869.44 and dividing this by 100 results in 5459. Finally we multiply the result by 2, because Hanover tabulates 2F, and we get 10918. That means that Hanover's variation of the moon's argument of latitude is very close to the modern value and is worth 483,201°.9994 in a period of 36525 days. Now F>2k * 180° and therefore the parity is even.

Table 2. Mean Movements of the Sun and Moon, the Molad, the Corrections and Supplements during Years of the Cycle

1. *Tikkunim*

   Let us examine the line with 18 years corresponding to 222 months.

   The correction is 222 * 0.045 782 978724 = 10.1638 hal = 10 hal 9.82/60, hence 10 hal 10/60 given by Hanover.

2. *Yitronot*, the differences between the lunar years, multiples of lunations and the tropical years.

   Generally the tropical years are longer than the lunar years. For example, if we consider the case of 18 years:

   18 tropical years, according to the length of Hanover, are: 6574.361 999 30 d.
   222 astronomical lunations:[39] 222 * 29.530 592 369 483 =
   6555.791 506 03 d.
   Difference 18.570 493 27 d = 18d 13h 747 ch.

   The only exception is the case of 8 years which are shorter than 99 lunations.
   8 tropical years, according to the length of Hanover, are: 2921.938 666 358 d.

38 Jean Meeus, Astronomical Algorithms, Willmann-Bell, chapter 45, p. 308.

39 According to the length of Hanover.

99 astronomical lunations:[40] 99* 29.530 592 369 483 = 2923.528 644 578 d.
Difference − 1.589 978 220 d = 1d 14h 172 hal.

At the creation of the world, the mean autumnal equinox was 15d 2h 235 hal. after the astronomical mean conjunction corresponding to *Beharad.* This mean conjunction followed *Beharad* by 42176 months * 0.152 609 92s = 6583.6399s =1h 49m 44s. This initial delay of 15d 2h 235hal must be added to the *yitronot* of the individual years (except the case of 8 years) and fractions of year, which increase the delay of the astronomical mean *tekufot.* On the contrary, the *yitronot* of the cycles, which bring the mean *tekufot* forward, must be subtracted.

3. The sun's mean anomaly

For example, after 18 years corresponding to 222 lunar months (astronomical months of Hanover) the variation of the sun's anomaly is:
(6555.791 506 03 / 36525) * 35,999°.043 792 3[41] = 6461°.3888 = 341°.3888 = 1228999.68". Hence 12290 given by Hanover.

4. The moon's mean anomaly

If we examine the line with 18 years, the variation of the moon's anomaly after 222 lunar months of Hanover is:
(6555.791 506 03 / 36525) * 477,198°.576 525[42] = 85,651°.3176 = 331°.3176 = 1,192,743".3600. Hence 11927 given by Hanover.
F > (2k + 1) * 180 and therefore the figure of parity is uneven.

5. The moon's argument of latitude

The argument of latitude corresponding to the line with 18 years is:
F = (6555.791 506 03 / 36525) * 483,201°.9994[43]= 86,728°.8587 = 240* 360+328°.8587 = 328°.8587 = 1,183,891"3200. Hence 2F = 11839 * 2 = 23678; corresponding to 10718 given by Hanover, after subtraction of 12960.

40 According to the length of Hanover.
41 The value adopted by Hanover.
42 See above, it is the value adopted by Hanover.
43 See above, it is the value adopted by Hanover.

Table 3. Mean Movements of the Sun and Moon, the Molad, the Corrections and Supplements during the Months

The values given for one month for the *yitronot* are those of one year divided by 12.

Table 4. Movement of the Sun's Anomaly during Days and Hours

The variation of the sun's anomaly for 29 days is given by:
(29 / 36525) * 35,999°.043 792 3 = 28°.5824 = 102,896.64". Hence 1029 given by Hanover.

Table 5. Movement of the Moon's Argument of Latitude during Hours

The variation of the moon's argument of latitude for 12 hours is given by:
483,201°.9994: (36525 * 2) = 6°.614 674 8720 = 23,812".8295. Hence 238 * 2 = 476 given by Hanover.

Table 6. The Quota of the Sun's Anomaly or the Equation of the Centre, and the Instantaneous Velocity of the Sun in Longitude (Variation of the Sun's True Longitude per Hour)

The Quota of the Sun's Anomaly

The anomaly of the sun and the moon varies between 0° and 360° or between 0 and 1,296,000". Hanover tabulates the anomaly in units of 100", from 0 until 12960. The area 0 until 6480 is read on the left column downwards; the quota of the anomaly is subtractive. The area 6480 – 12960 is read on the right column upwards; the quota of the anomaly is additive.

The quota of the sun's anomaly, or the equation of the centre, represents the difference:
$\Lambda - L$ i.e., the difference between the true longitude and the mean longitude. The study of the elliptic movement allows writing:[44]

$C = 1°.914\,600 \sin M + 0°.019\,993 \sin 2M + 0°.000\,290 \sin 3M + \ldots\ldots\ldots\ldots$
The sun's true longitude is $\Lambda = L + C$.

44 Equation of the centre for 2000 according to Meeus, Willmann-Bell 1991, chap. 24: Solar Coordinates.

We have already mentioned that in ancient astronomy and even in the astronomy of the eighteenth century, the anomaly is calculated with regard to the sun's apogee and therefore the sign of C changes: the equation $\Lambda - L = C$ becomes $\Lambda - L = - C$ in ancient astronomy and even in modern astronomy of the eighteenth century.

The equation of the centre given by Hanover is not very precise; it is even less precise than the quota of the anomaly given in volume 2 according to Maimonides, following the ancient astronomy of Ptolemy. It is difficult to understand how, in the eighteenth century, Hanover gives an equation of the centre of 2.06° for M=90° and 270°. In the following comparative table M is the anomaly according to the modern definition, referring to the perigee; it is expressed in degrees. The equation of the center is positive for M < 180°. It is expressed in seconds of arc: 1" = 0.0002778°.

$M = \Lambda - L$	Meeus (1900)[1]	Hanover (1756)	Hanover (ancient astronomy)	Lalande (1764)
0°	0	0	0	0
10°	1225.2	1318	1260	1229
20°	2410.81	2592	2520	2418.3
30°	3518.74	3784	3660	3529.7
40°	4513.86	4859	4740	4527.8
50°	5365.17	5770	5580	5381.8
60°	6046.92	6503	6300	6065.6
70°	6539.28	7029	6780	6559.3
80°	6828.89	7337	7080	6849.7
90°	6909	7425	7140	6930.1
100°	6779.44	7281	7020	6800.1
110°	6446.32	6926	6660	6465.8
120°	5921.65	6350	6060	5939.5
130°	5222.7	5605	5340	5238.4
140°	4371.37	4689	4500	4384.4
150°	3393.42	3641	3480	3403.5
160°	2317	2490	2400	2328.8
170°	1175.69	1262	1200	1179.3
180°	0	0	0	0

The solar equation of the Center. M = 10° corresponds for Hanover and Lalande to an anomaly of 190°. One can observe the very good coincidence between the values of Lalande and the modern values. The values given by Hanover are less precise. Engineer Eran Raviv got a perfect coincidence between the values given by Hanover and those derived from the theoretical formula:
$C = (2e - 0.25e^3) \sin M + 2.5\, e^2 \sin M * \cos M$ for an eccentricity of $e = 0.017995 \sim 0.018$ instead of the correct value of 0.01680 adopted by Lalande[45]. The greatest equation of the sun is 7425"= 2.06° (Hanover) instead of 1° 55' 31.6" (Lalande). The values of Hanover are worse than those adopted by al-Battani nearly 8 centuries before. The approach adopted by Hanover remains a conundrum. M is given in degrees and C in seconds of arc.

Instantaneous Velocity of the Sun in Longitude

$\Lambda = L + 1°.914\,600 \sin M + 0°.019\,993 \sin 2M + 0.000\,290 \sin 3M$...........[46]
or in radians:
$\Lambda = L + 0.033\,4160\,74 \sin M + 0.000\,348\,94 \sin 2M + 0.000\,005\,06 \sin 3M$........

If we want to express the velocity in seconds of arc per hour we need to know:
$dL / dt = 36000°.769083/36525 = 0.985\,647\,360$ °/day or 147.8471 "/h. and
$dM / dt = 35999°.050030 / 36525 = 0.985\,600\,281$ °/day or 147.8400 "/h.
The instantaneous velocity of the sun on the ecliptic is thus:
$d\Lambda / dt = 147.8471''/h + 4.9402 \cos M + 0.1032 \cos 2M + 0.0022 \cos 3M$
When the sun is at the perigee the angular velocity is maximal: 147.85 + 4.94 = 152.79"/h.

When the sun is at the apogee, the velocity is minimal: 147.84 – 4.94 = 142.9»/h. Hanover rounds off at 143"/h, 148"/h and 153"/h. The same procedure allows calculating the angular velocity of the true longitude and true anomaly for any value of M. It is likely that Hanover calculated the velocity by another procedure, using his favorite method of the finite differences. For example, when M = 0 and the sun is at its perigee, for Hanover the anomaly is 180°, the quota of the anomaly is 0°.

45 The comparison of table 6 with the table of figures obtained by the theoretical formula allowed Eran Raviv to correct some misprints in Table 6. For the anomaly of 1°: 127 instead of 172 (as indicated in fact in the erratum), for 59°: 6284 instead of 6254 and for 144°: 6845 instead of what could be erroneously read as 6815 because the 4 is very weak.

46 Astronomical Formulae for Calculators, Jean Meeus, Willmann-Bell 1982, chapter 18: Solar Coordinates, p. 80. Astronomical Algorithms, Jean Meeus, Willmann-Bell 1991, chapter 24, Solar Coordinates, p. 152.

When M = 1° (for Hanover the anomaly is 181°), then the quota of the anomaly is 133". Thus when the mean anomaly of Hanover increases from 180° to 181° the true anomaly increases from 180° to 181.0369°. The true velocity is thus the mean velocity multiplied by 1.0369 or 147.8 * 1.0369 = 153"/h.

Table 7. The Moon's Quota of the Anomaly at Mean Conjunction or Opposition and the Moon's Angular Velocity of the True Longitude

The movement of the moon is much more complicated. The quota of the anomaly corresponding to Λ' – L' i.e. the difference between the true longitude and the mean longitude, includes in addition to the equation of the centre, different perturbations, some of them having a special name. The most important of these perturbations is the evection which was already detected by Hipparchus of Nicea in the second century BCE, but Ptolemy, in the second century, was the first to formulate the law of its time dependence. We have the following relation between Λ', the true moon's longitude and L' the mean moon's longitude:[47]

Λ' = L' + 6°.288 774 sin M' + 1.274 027 sin (2D – M') + 0°.658 314 sin 2D + 0°.213 618 sin 2M' – 0°.185 116 sin M – 0°.114 332 sin 2F + 0°.058 793 sin (2D – 2M') +..........

The same relation, written in radians, gives:

Λ' = M' + 0.109 759 812 sin M' + 0.022 235 966 sin (2D – M') + 0.011 489 747 sin 2D + 0.003 728 337 sin 2M' – 0.003 230 884 sin M – 0.001 995 470 sin 2F + 0.001 026 131 sin (2D – 2M') + ...

In order to calculate the derivative of this relation, we need the following data:

dL' / dt = 1976.4595"/h.

dM' / dt = 1959.7489"/h.

dD / dt = 1828.6124"/h

dF /dt = 1984.4025"/h

d (2D – M') /dt = 1697.4758"/h.

d (2D – 2M') / dt = – 262.2932"/h.

We find then, deriving the former relation and adopting "/h as a unit of angular velocity:

dΛ' / dt = 1976.4595"/h + 215.1017 * Cos M' + 37.7450 * Cos(2D – M') + 42.0260 * Cos 2D + 14.6132 * Cos 2M' – 0.4777 * Cos M – 7.9196 * Cos 2F – 0.2691 * Cos(2D – 2M') + ...

47 Astronomical Formulae for Calculators, Jean Meeus, Willmann-Bell 1982, chapter 30: position of the moon, p. 149.

At the time of the syzygie or opposition, D = 0 and if M' is equal to 0, then Λ' will be maximum. The maximum value of Λ' is about 1976.4595 + 215.1017 + 37.7450 + 42.0260 + 14.6132 – 0.2691 ~ 2285"/h. Similarly the minimum value is reached for M' = 180° and is about 1780"/h. If we consider only the first perturbation term we have: Λ' max. = 2191.56"/h and Λ' min. = 1761"/h.

Hanover probably used a simplified equation of the center.

The eccentricity of the moon's trajectory is today about 0.0549 and the simplified equation of the centre given by the theory of the elliptic movement is then:

C' = 0.01098 sin M' + 0.0038 sin 2M', C' being calculated in radians; or in degrees:

C' = 6°.2887 sin M' + 0°.2159 sin 2M'.

The evection is given by Ev = 1°.2739 sin (2D – M'), where D = L – L' is the mean elongation. At the conjunction D~ 0° and Ev = – sin M', it diminishes the quota of the anomaly to 5°.0148 sin M' + 0°.2159 sin 2M'. Hanover gives 4.99° for M' = 90 and 270° which is a very good approximation.

The variation of the moon's true longitude per hour or the angular velocity of the moon's true longitude could then have been calculated by Hanover as follows: Λ'= L' + C'.

dΛ '/dt = dL '/dt + dC'/ dt

dL'/dt is the angular velocity of the mean longitude, and its value[48] is 481,267°.881342 /36525= 13°.176396477°/day or 1976.4595 "/h. If we express C' in radian and neglect the second term: C' = 0.0875 sin M' and therefore dC '/dt = 0.0875 cos M' dM '/dt.

Where:

dM '/dt = 477198°.8676313/ 36525 = 13.0650°/day or 1959.7489"/h.[49] We see that the angular velocity of the mean anomaly and the mean longitude are respectively 1959.75"/h and 1976.46"/h, and are not very different[50] from each other; they differ by less than 1% and therefore the angular velocity of the true longitude and the true anomaly are also very close.

When the moon is at the perigee the angular velocity is maximal: 1976.46 + 0.0875*1959.75 = 2148"/h.

48 According to the modern value, which does not differ appreciably from Hanover's value.

49 This is again the modern value, which does not differ appreciably from Hanover's value.

50 The difference between the angular velocity of the longitude 1976.4595"/h and the angular velocity of the anomaly 1959.7489"/h is 16.7105"/h; it represents the angular velocity of the apogee and perigee of the lunar orbit. In the case of the sun the difference between the angular velocity of the longitude and the anomaly is only 0.0071"/h.

When the sun is at the apogee, the velocity is minimal: 1976.46 – 0.0875*1959.75 = 1805"/h.
Hanover rounds off the angular velocity to 1814"/h, 1966"/h and 2157"/h.

The same procedure allows one to calculate the angular velocity of the true longitude for any value of M. It is likely that Hanover calculated the velocity by another procedure, using his favorite method of the finite differences. For example when M'= 0 the moon is at its perigee, thus for Hanover the anomaly is 180°, the quota of the anomaly is 0°.
When M' = 1° and for Hanover the anomaly is 181°, then the quota of the anomaly is 332". Thus when the mean anomaly of Hanover increases from 180° to 181° the true anomaly increases from 180° to 181.0922°. The true velocity is thus the mean velocity multiplied by 1.0922 or 1966 * 1.0922 = 2147"/h. In fact, there is a lack of precision and coherence in the velocities given by Hanover.

It is nevertheless the velocities of the moon and the sun on the ecliptic which we are searching for, thus we must use the true angular velocities of the moon's and sun's longitude. It is thus strange that Hanover did not use 1976"/h as the mean angular velocity of the moon's longitude instead of 1966"/h.

Principle of Utilization of Tables 6 and 7

Hanover's tables 1 to 5 are based on the mean movements of the sun and moon. Because of the eccentricity of the orbits, the sun may be 1°.9 (maximum value of the sun's equation of the centre) on either side of its mean position and the moon 6°.3 (the maximum value of the moon's quota of the anomaly). Moreover, there are periodic perturbations in the moon's longitude. However, at the new and full moon D = L – L'~ 0 or 180°, the evection and other perturbation terms reduce the moon's maximum deviation from 6°.3 to 5°.4. Therefore, the relative positions of the two celestial bodies may vary 1°.9 + 5°.4 = 7°.3 from the mean value near the conjunction or the opposition. As the hourly motion of D = L – L' is 0°.51, the maximum time interval Δt between the mean new (or full) moon and the new true (or full) moon will be 7°.3 / 0°.51 = 14.3 hours. This explains why Table 5 is calculated with an entry for maximum 14 hours.

Now one finds at the mean conjunction the mean anomaly of the sun M = L – Γ – 180° and then through Table 6 the quota of the anomaly Λ – L, i.e. the distance between the true and the mean sun. One also finds the mean anomaly of the moon and then through Table 7 the quota of the anomaly of the moon Λ' –L'. But at the mean conjunction L = L', and therefore we know Λ –Λ', the distance between true sun and true moon. Now if we consider the position of the true moon and the true sun, there are two possibilities:

a) *The sun's true longitude is greater than the moon's true longitude.*
The true conjunction will occur Δt after the mean conjunction. During this time Δt the moon must catch up with the sun. As the moon's velocity is about 13 times the sun's velocity, the moon will, during this span of time, cover the distance between the moon and the sun + the little distance covered by the sun. This is the reason why Hanover takes the instantaneous velocity of the moon at half-way of the distance between true moon and true sun at the moment of the mean conjunction.

b) *The true moon's longitude is greater than the true sun's longitude.*
In this case the moon has already outrun the sun and therefore the true conjunction was Δt before the mean conjunction.

With the instantaneous angular velocity of the moon at half-way the distance between the true moon and the true sun, and with the angular velocity of the sun we find by subtraction the relative angular velocity. The quotient of the distance by the velocity gives the time Δt which must be added to or subtracted from the time of the mean conjunction to get the true conjunction. The next step is then to find the argument of latitude F of the true moon.
F (true moon) = F (mean moon) + $(\Lambda' - L') + \Delta F(\Delta t)$.
Or: $2 * F$ (true moon) = $2 * F$(mean moon) + $2\,(\Lambda' - L') + 2 * \Delta F(\Delta t)$, where the first term has been calculated through the first tables, the second term is found through Table 7; it is twice the quota of the moon's anomaly divided by 100 and the last term is calculated through Table 5 for the span of time Δt between mean and true conjunction.

Numerical Examples

Hanover considers in his first example, to which we will limit ourselves, the conjunction of Adar II of the year 5497 in Hanover, corresponding to Friday, March 1, 1737.

1. Calculation of the *molad*. The molad of Adar II was 7 – 1 – 650.

2. Calculation of the corrected *molad* = mean conjunction.

   The number of elapsed years between the current year 5496 and 3411 = 2085 =

109 * 19 + 14 years. The total correction is 1h 101 hal. The mean conjunction is thus 7-0-549 in Jerusalem and 6-22-464 in Hanover. The modern mean conjunction calculated with the Table of Meeus[51] gives the mean conjunction at 16h 03m in Hanover, about 20 minutes before.

3. Calculation of M and M', the sun and moon's mean anomalies at the mean conjunction.

   According to the procedure of Hanover, based on the fact that the *molad* of Adar II was preceded by 289 cycles of 19 years, 5 years and 6 months, we find M = 60530 and M' = 37808. As 360° = 1,296,000" or 12960 ("/100) we subtract the greatest possible multiple of 12960 and find M = +8690 and M' = +11888.

4. The quota of the sun's anomaly.

   The sun's mean anomaly is 8690. For M = 8676 the quota is +6563
   For M = 8712 the quota is +6624
   Difference of M = 36 and difference of the quota is 61.
   Thus for M = 8690, the quota is 6563 + 61 *(14/36) = +6587.
   The angular velocity of the sun is 151"/h.

5. The quota of the moon's anomaly.

   The moon's anomaly is 11888. For M' = 11880 the quota is +8588.
   For M' = 11916 the quota is +8324.
   Difference of M'= 36 and difference of the quota is 264.
   Thus for M' = 11888 the quota is +8588 – 264 *(14/36) = +8529. The moon's velocity is 1832"/h. At the moment of the mean conjunction the mean sun and mean moon coincide; the true sun is ahead by 6587" and the true moon is ahead by 8529". The distance between true sun and true moon is 8529 – 6587 = 1942", the moon being ahead of the sun by 1942". The velocity of the moon is about 13 times the sun's velocity. Therefore the coincidence of sun and moon occurs near the position of the true sun at the moment of the mean conjunction. The greatest part of the distance of 1942" between sun and moon at the time of mean conjunction is covered by the moon. The mean anomaly of the moon at the time of mean conjunction is 11888 and the true anomaly of the moon

51 Meeus Jean, Syzygies Tables, Kessel-Lo 1963.

at the same moment is 11888 + 85 = 11973. The mean value of the moon's true anomaly during the time used by the true moon to cover the distance of 1942" is 11973 – (19.42/2) = 11963. The mean anomaly at the same moment is 11963 – 85 = 11878 and the corresponding velocity of the moon is 1832.[52] The relative velocity of the two bodies is 1832"/h – 151"/h = 1681"/h. The true conjunction was 1942/1681 = 1.06 h = 63.6 m = 1h 168 hal. before mean conjunction, because at mean conjunction the true moon had already outrun the sun by 1942".

6. Calculation of the true conjunction.

   The mean conjunction was at 7-0-549, the true conjunction was 0-1-168 before at 6-23-381 in Jerusalem or 6-21-296 in Hanover. According to the table of Meeus, we find a perfect coincidence: Friday, March 1, 15h 16m Hanover mean time.

7. Calculation of the following mean opposition, on Saturday, March 16, 1737.

   We depart from the mean conjunction (corrected *molad*), to which we add 0-18-396, the modulo of 14-18-396[53] with regard to 7. The mean opposition was thus 7-18-945.

8. Calculation of M and M', the sun and moon's mean anomalies, at the moment of the mean opposition.

   At the moment of the mean conjunction (*molad* corrected i.e. *molad* minus correction).

   We have found M = 8690 and M'= 11888. We add the variation of the anomaly for a half month: M = 8690 + 524 = 9214 and M' = 11888 + 6945 = 18853 = 18853 – 12960 = 5873.
   From Table 6 we find the quota of the sun's mean anomaly C(M) = +7239 and from Table 7 we find the quota of the moon's mean anomaly C' (M') = – 5503. The distance between the two bodies is thus 7239 + 5503 = 12742. The moon's

52 Thus this little correction replaces the moon's mean anomaly at the moment of mean conjunction of 11888 by the moon's mean anomaly at half of the covered distance of 1942" corresponding to the distance between mean conjunction and true conjunction. At this moment the mean anomaly is 11888 – 10 = 11878. This allows calculating the moon's velocity with more precision.

53 The half of 29 – 12 – 793, the length of one month.

mean anomaly at the half of the covered distance between the mean moon at mean opposition and true moon at mean opposition is 5873 – (127, 42/2) = 5873 – 64 = 5937. The corresponding moon's velocity is 2150"/h while the sun's velocity is 150"/h and the relative velocity is 2000"/h. The span of time, counted from the mean opposition allowing the moon to catch up to the sun is 12742/2000 = 6.37 h or 6h 401 ch. The true opposition is then 7-18-945 + 0-6-401 = 1-1-266 in Jerusalem and 7-23-181 in Hanover corresponding to Sunday, March 17, 1737 at 17h 10m.

9. Calculation of the moon's argument of latitude at the moment of true conjunction in order to check the possibility of a solar eclipse.
   At the beginning of Nissan 1737, 5496 years have elapsed from *Beharad*, corresponding to 289 cycles 5 years and six months. Twice the argument of latitude, at the moment of the mean conjunction, is found to be 25248 and the parity is 9. In order to calculate the argument of latitude at the true conjunction we apply the relation examined above:
   2 * F(true moon) = 2 * F(mean moon) + 2 (Λ' – L') + 2 * ΔF(Δt),
   2F(true moon) = 25248 + 2*8529/100 + 2ΔF(-1h 168ch)[54]= 25248 + 171 – 46 = 25373 = 12413 after subtraction of 12960. The figure of parity which was 9 becomes 10.
   2F = 12413 means that the moon is near one of the nodes because 12960 – 12413 = 547 < 1150. According to the rules given in Table 8, there is a solar eclipse at the moment of true conjunction and the sun is eclipsed in its upper part because the moon is north of the sun. Indeed the true conjunction occurred at 15h 16m in Hanover, and the solar eclipse was visible.

10. Calculation of the moon's argument of latitude at the moment of the true opposition in order to check the possibility of a lunar eclipse.
   We depart from twice the argument of latitude at the mean conjunction, and add twice the argument of latitude for a half month, i.e. 1104. We obtain at the mean opposition:
   2 * F = 25248 + 1104 = 26352, with a figure of parity equal to 9 + 1 = 10. Now at the true opposition:
   2 * F(true moon) = 2* F(mean moon) + 2 (Λ' – L') + 2 * ΔF(Δt),
   2 * F(true moon) = 26352 – (2 * 5503) /100 + 2ΔF(6h 401 ch) = 26352 – 110 +

54 Table 5.

253 = 26495. After subtraction of 25920, corresponding to twice 360°, we get 575 with a parity figure of 10 + 2 = 12. As 380 < 575 < 864 we have a partial lunar eclipse and since the figure of parity is even, the moon is north of the sun. Nevertheless this partial lunar eclipse of its inferior part happens around 17h 10m Hanover mean time and the eclipse could not be seen because the sun had not yet set at this hour, and the moon was not yet visible at this time.

11. Calculation of the mean equinox

Thanks to the *yitronot*, we calculate the distance of the mean equinox with regard to the mean conjunction or corrected *molad*. This *molad* was preceded by 289 cycles, 5 years and 6 months. We add to the radix 15-2-235, representing the delay of the autumnal mean equinox of *Beharad* with regard to the corrected *molad* of *Beharad*, the *yitronot* of the years and months which are longer than the lunar years, and this gives the first sum +(45–9–14).[55] We add the *yitronot* of the different cycles, which are longer than the tropical years, and we get –(24–12–743); this gives the second sum.[56] We subtract it from the first sum and get 20-20-351, representing the delay of the mean equinox with regard to the *molad* of the seventh month, in our case Adar II. The mean conjunction (corrected *molad*) of Adar II 5497 was 7-0-549 and the equinox was 27-20-900 or 6-20-900. It corresponds to Friday, March 22, 1737.

12. Calculation of the true equinox

The sun's mean anomaly at the moment of the mean conjunction of Adar II was 8690. We add to it the variation of the sun's anomaly during 20 days i.e. 710, during 20 hours, i.e. 30 and during 351 ch, i.e. ~0, in total 740, the sun's mean anomaly at the moment of the mean equinox is then 8690 + 740 = 9430. The corresponding sun's quota of the anomaly is +7373 and the angular velocity of the sun is 149. In other words, at the moment of the mean equinox, the distance between the true sun and the mean sun is 7373". The time necessary for the sun to cover this distance is 7373 / 149 = 49.4832 = 49h 522 ch. At the moment of the mean equinox the true sun was in advance by 7373" with

55 These *yitronot* are related to the spans of times longer than the lunar months. This first sum, given in days, hours and halakim, represents the delay of the *tekufa* after the corrected *molad*.

56 These *yitronot* are related to the spans of time shorter than the lunar months. The second sum, given in days, hours and halakim, represents a span of time before the corrected *molad*.

respect to the mean sun, the true equinox thus preceded the mean equinox and was on 6-20-900 – 2-1-522 = 4-19-378 in Jerusalem and 4-17-293 in Hanover, corresponding to Wednesday, March 20, 1737, at 11h 16m or 10h 55m G.M.T.

13. Comparison with more precise data[57]

If we compare the results of Hanover with the tables of Meeus, we get the following comparison.

At the mean conjunction: M (Hanover) = 8690 M +180° (Meeus) = 8677
M' = 11888 M' +180° = 11908
2F = 12288 2F = 2*170.89° = 12304

indeed 1° = 3600' = 36.

Mean conjunction: 6-22-464 in Hanover or 16h 26m.

Meeus: Friday, March 1, 1737 at 16h 03m in Hanover.

True conjunction: Hanover: 6-21-296 in Hanover or 15h 16m.

Meeus: Friday, March 1, 1737 at 14h 57m in Hanover.

Mean equinox: Hanover: 6-20-900

True equinox: Hanover: 4-17-293 in Hanover or 11h 16m.

Meeus: Wednesday, March 20, 1737 at 14h 1m in Hanover.

Conclusions and Acknowledgements

The book *Luhot ha-Ibbur*, printed in 1756, was aimed at well-read Jewish people, who were not able to find and consult specialized books in German. It is even likely that a similar book did not exist in German. It was not common to find a book, based on astronomical and reliable data, that was written for laymen. This book can be compared to the "Syzygie Tables"[58] which allow the calculation of true conjunctions and oppositions, and check the occurrence of solar or lunar eclipses. All the books of Meeus depart from the same principle: writing astronomical books at a professional level, with numerical data adapted for practical use, aimed at laymen and lovers of astronomy. The *Luhot ha-Ibbur* were constructed on the basis of the Jewish calendar with remarkable precision, rigorous logic and order, justifying Hanover's reputation as an extraordinary skilled calculator. The only shortfall that could be suggested is the absence of explanations and justifications.

57 According to contemporary data.
58 Jean Meeus, 1963.

The author was aware of the problem and he intended to write a third part to his book for that purpose. Nevertheless, the examples are very detailed and complete and allow readers to learn the calculation methods.

The tables of the solar and lunar mean movements are calculated with the highest precision. Nevertheless, the length of the moon's synodic lunation and the length of the tropical year are slightly different from modern values and less good than the data adopted during the same epoch by Tobias Mayer and Lalande.

Hanover's tables take into account a rough approximation of the equation of the centre for determining the true position of the sun, and only the equation of the centre and the evection for determining the true position of the moon.

In Table 6, relative to the sun, we observed the lack of precision of the quota of the anomaly (equation of the centre) which reaches a maximum of 2°.06[59] instead of 1°; 59' adopted by Al-Battani[60] and the modern value of 1°; 55'[61] adopted by Lalande.

In Table 7, relative to the moon, the moon's velocity in longitude is also unjustifiable. The mean velocity of 1966"/h is compromised between 1959.75"/h (the mean velocity of the moon's anomaly) and 1976.46"/h (the mean velocity of the moon's longitude). Similarly Hanover's minimum and maximum velocities cannot be justified.

59 See Table 6: 7426 for a solar anomaly of 3276 = 91°. This value is much too high; it is nearly the value of Ptolemy!

60 In about 980 CE. This value was adopted by Maimonides in *Hilkhot Kiddush ha-Hodesh*.

61 Lalande gives 1° 55' 31.6" and an eccentricity of 0.01680207 in his Astronomy, tome 2, n° 1266, Paris 1764, 1771 and 1791 (dates of the three editions).

APPENDIX

The Timetable of Hanover (1766)[62]

This little document, on one sheet of paper, deserves much attention because it represents a real revolution in Jewish life regarding the calculation of *halakhic* times throughout the day, and more specifically the beginning and end times of the Sabbath. It is indeed the first printed document calculating these times on the basis of a fixed depression of the sun under the horizon throughout the year. The time is expressed in true time. The table was established for a latitude of 52.5°. The refraction adopted in the eighteenth century was 0°; 32'[63]. The obliquity of the ecliptic was probably 23°; 29'.[64] Furthermore in the eighteenth century sunrise and sunset were moments when the apparent position of the centre of the sun is on the horizon, i.e. when the solar depression is 0°; 32'.[65] On this basis I have calculated that Hanover considered a solar depression of

- 8°; 05' for the time "*mishe'yakir*" משיכיר, which he calls "*alot ha'shachar*."
- 0°; 32' for sunrise and sunset
- 7°; 10' [66] for "*tzet ha'kochavim*" (appearance of the stars)

This table was acclaimed by some rabbinical authorities of Western Europe. Rabbi Tsvi Hirsh Levin of Berlin (1721–1800) and his son Solomon Hirshel (1762–1842) used it to construct a more detailed liturgical horary based on the calculation of long temporary hours, which assumed that the religious day begins with a solar depression of 8°; 05' and ends with a depression of 7°; 10'.[67] Rabbi Nathan Adler (1741–1800) used the table of Hanover and adapted it to his town of Frankfurt without taking into account the change of latitude. Rabbi

62 This table is reproduced on p. 525 of *Ha-Zemanin ba-Halakha*, P. Benish, 1996.

63 Instead of: 0°; 34' today.

64 The more accurate value of 23°; 28' determined by Bradley was not yet widely known.

65 The modern definition of sunrise and sunset is the apparent passage at the horizon of the upper limb of the sun. It corresponds to a solar depression of 0°; 50 '. This definition is the same as the *halakhic* sunrise or sunset.

66 This value became the rule until the second half of the twentieth century (tables of Berthold Cohn, Calendar of Bloch) when more stringent customs imposed themselves: depression of 8 and even 8.5°.

67 The first page of this table is reproduced on p. 526 of *Ha-Zemanim ba-Halakha*, Benish 1996. This table presents a slight asymmetry with regard to noon. The principle of calculating the long temporary hours has evolved with time. The manuscript of these tables is in the Library of the Jewish Theological Seminary.

Moses Schreiber (1762–1839) received a copy of his teacher's table and used it in Mattersdorf and Presburg.[68]

The principle adopted by Hanover to work on the basis of a constant solar depression in order to calculate the beginning and end *halakhic* times of each day, as well as the Sabbath, was slowly adopted in Eastern Europe during the nineteenth century; today it is an accepted fact.

The First Appearance of Any Given Molad

1. Since the completion of my article in *B.D.D.* 28, I edited Hanover's manuscripts and among them "*Sefer Tekhunat Ha-Shamayim Ha-Arokh*"– ספר תכונת השמים הארוך – where I found at the end of that book that Hanover improved the procedure of finding the first appearance of a given *molad*. Instead of our modern formula, Hanover constructed two very convenient tables.

2. Already in the first half of the fourteenth century, Rabbi Isaac Israeli proposed a solution to this problem[69] but it was less elegant and more difficult. The solution was based on two tables: the first table, לוח ג', gives the *molad* of the first 1080 months of the Jewish era. The first *molad* of the table is 2–5–204 and the last *molad* is 3–6–204.

   Indeed $[1080 * (1 - 12 - 793)]_{181440} = 27000 = 25920 + 1080 = 1d + 1h$.
   Thus after 1080 months the *molad* is 1d 1h up.
   The second table, לוח ד', gives the *molad* at the beginning of the first 168 cycles of 1080 months. After each cycle the *molad* is 1d 1h up. After 168 cycles the final *molad* is again the initial *molad*. Indeed 168 * (1d 1h) = 175d = M7.

3. Hanover's discovery of the integer 74377 was therefore not such an achievement. Hanover had the merit to determine after which number of months the *molad* 2–5–204 is 1 *helek* up and becomes 2–5–205. He probably used the method of Israeli.

   In לוח ג' we find the *molad* ending with 205 *halakim*. This *molad* occurs after 937 months; it is 1–9– 205. Indeed $[31524 + 937 * 39673]_{181440} = 9925 = 1 - 9 - 205$.

68 The adaptation of Hanover's table by these two rabbis, without taking into account the important changes of latitude, is notably the subject of a paper published by engineer Yaakov Loewinger of Tel Aviv in *ha-Maayan* Teveth 5772 (2012) n° 200, pp. 23-50 and entitled: על זמן בין השמשות, ועל מילה בשבת של תינוק הנולד סמוך לצאת השבת.

69 See *Yessod Olam, ma'amar* V, chap. 4 and at the end of the book לוח ג' ולוח ד'.

We must add 20h in order to find the *molad* 2–5–205.

In לוח ד' we see that after 68 cycles of 1080 months the initial *molad* is 20h up. Indeed

$[68 *(1d + 1h)]_{7d} = 20h$. Thus after 68 * 1080 + 937 = 74377 months the initial *molad* 2 –5–204 became 2–5–205.

It appears that the finding of Hanover's number, using Israeli's algorithm did not present a major difficulty. Hanover's great originality was to look for the number of months after which the *molad* is 1 *helek* up, and then to propose a simple and elegant solution by constructing a table giving the number of months necessary to result in an increase of the *molad* by different multiples of 1 *helek*.

4. Recently while editing the present paper, I found at the end of Hanover's manuscript *Tekhunat ha-Shamayim ha-Arokh*,[70] the following three tables and an example, without any explanation or justification. The process is now easy to understand and the elegance and rapidity of the procedure are evident.

לוח המולדות

חודשים	חלקים	חודשים	חלקים
172060	תש	74377	א
170720	תת	148754	ב
169380	תתק	41691	ג
168040	תתר	116068	ד
154640	ב אלפים	9005	ה
141240	ג אלפים	83382	ו
127840	ד אלפים	157759	ז
114440	ה אלפים	50696	ח
101040	ו אלפים	125073	ט
87640	ז אלפים	18010	י
74240	ח אלפים	36020	כ

70 See http://www.ajdler.com/jjajdler/hanover/ pp. 134-136.

60840	ט אלפים	54030	ל
47440	י אלפים	72040	מ
94880	כ אלפים	90050	נ
142320	ל אלפים	108060	ס
8320	מ אלפים	126070	ע
55760	נ אלפים	144080	פ
103200	ס אלפים	162090	צ
150640	ע אלפים	180100	ק
16640	פ אלפים	178760	ר
64080	צ אלפים	177420	ש
111520	ק אלפים	176080	ת
41600	ר אלפים	174740	תק
		173400	תר

לוח מספרי הגירעון
181440
362880
544320
725760
907200

מספר החודשים

מחזורים	חודשים	שנים	חודשים
א	235	א	12
ב	470	ב	24
ג	705	ג	37
ד	940	ד	49
ה	1175	ה	61
ו	1410	ו	74
ז	1645	ז	86
ח	1880	ח	99
ט	2115	ט	111
י	2350	י	123
כ	4700	י"א	136
ל	7050	י"ב	148
מ	9400	י"ג	160
נ	11750	י"ד	173
ס	14100	ט"ו	185
ע	16450	ט"ז	197
פ	18800	י"ז	210
צ	21150	י"ח	222
ק	23500		
ר	47000		
ש	70500		
ת	94000		
תק	117500		
תר	141000		
תש	164500		

אם תרצה לידע באיזה חודש או שנה או מחזור יהיה או היה מולד הנתון?
תעשה כך: מן מולד שבידך תגרע ב ה ר"ד והנשאר תעשה לחלקים. וקח מלוח העליון – בעמוד הקודם – מספר החדשים העומדים לנגד החלקים שבידך ותחברם יחד ומהכלל תגרע מלוח הגירעון המספר שאתה יכול לגרוע ועם הנשאר לך אל לוח [התחתון] – בעמוד זה – וקח המחזורים והשנים [והחודשים העומדים] נגד המספר הנשאר וליוצא השנה והחודש שבו יהיה או היה מולד הנתון.
כגון שתרצה לידע באיזה שנה ובאיזה חודש יהי או יהיה מולד ד – י"ט – פ"ו ? תגרע ב – ה – ר"ד ונשאר ב – י"ג – תתקס"ב. תעשה לחלקים ויוצא 66842. וקח מלוח העליון המספרים העומדים ויהיה מולד הנתון מחודש [ניסן אחר] שעברו רי"ח מחזורים ט"ז שנים וז' חודשים ר"ל חודש ניסן משנת [4159].[71]

51434	ולך אל לוח התחתון	103200	60000
47000	ותגרע ל-ר' מחזורים	101040	6000
4434	נשאר	170720	800
2350	ל-י' מחזורים	72040	40
2084	נשאר	148754	2
1880	ל-ח' מחזורים	595754	66842 סך הכל
204	נשאר	544320	תגרע מלוח הגרעונים
197	ל- ט"ז שנים	51434	ונשאר
7	ונשאר		

We would like to know when the *molad* (4) – 19 – 86 occurred for the first time.
(4) – 19 – 86 – (2) – 5 – 204 = 2 – 13 – 962 = 66842 hal.
From the first table we deduce that this happened after 595754 months. But we know that the *molad* remains the same after a multiple of 181440 months. This *molad* was thus already reached after 51434 months. From the third table we deduce that 51434 months correspond to 200 cycles + 10 cycles + 8 cycles + 16 years + 7 months. The 17th year is a leap year and it leads us to Nissan 4159.

71 This year is a leap year and the eighth month is Nissan.