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The Period of 689,472 Years in the Jewish Calendar and Its Applications in Frequency and Probability Problems

The central topic of this paper is the study of the period of the Jewish calendar. After a period of 36288 cycles of 19 years or 689,472 years, the initial *molad* of *Beharad* recurs a second time at the head of a year of rank 1 of a 19-year cycle. This day is the second day of the week and corresponds to Tishri 1, 689,473 AMI. It also corresponds to Monday, November 4, 685,720 of the Gregorian calendar. The loop is then looped and a new cycle begins again. The Jewish calendar is thus periodic but the period is 689,472 years and the historic part of the calendar is only a very little part of this period.

In the first section we describe an easy method for calculating the Jewish calendar and converting a Jewish date into a civil date by using the Julian day.

In the second section we examine thoroughly the period of the Jewish calendar. We show that there is a correlation between the rank of year in the 19-year cycle and the last digit (the right digit) of the *molad* of that year.

Further we show that when we consider the *molad* of the different years, the probability of occurrence of the figures 0, 1, 2 etc until 9 in the last digit of the different years' *moladot* is not constant. The probability of occurrence of 3 or 8 in the last digit is 7.89% while the probability of occurrence of another figure – 0, 1, 2, 4, 5, 6, 7 or 9 – in the last digit is 10.53%.

It has always been admitted that all the *moladot* of the years have the same probability of occurrence. Therefore it was considered that there is a proportion between the frequency of the use of a certain *dehiyah* or type of year, and the width of the corresponding area of its *moladot*. This principle is only approximately true, but the approximation is sufficiently good and people have considered this principle as exact. We show that the exact calculation of the frequency of the use of a type of *dehiyah* or type of year requires a detailed counting during the period of the Jewish calendar.

In the fifth section we explain how it is possible to calculate very simply when any given *molad* occurs for the first time. In the sixth section we explain how it is possible to calculate the four or three occasions when any given *molad* becomes the *molad* of a year.

This paper provides a deeper understanding of the Jewish calendar.

I. EASILY CONVERTING A JEWISH DATE INTO A CIVIL DATE BY USING THE JULIAN DAY

The classical methods for converting a Jewish date into a civil date are long and dull. The principle rests on the calculation of Samuel's *tekufah* of September with regard to Tishri 1 and on the fact that the *tekufah* of Tishri always falls on September 24 in the Julian calendar. Louis A. Resnikoff[1] described an algorithm based on the same principle applicable in pocket calculators. Another method of computation makes use of the formula of Gauss[2] giving the date of Nissan 15 in the Julian calendar.[3]

We propose here a simple method in which we calculate the *molad* as a moment of the week and as a precise moment in history thanks to the Julian day. The method is conceptually very simple, but it must, however, be applied with care and precision.

Let us consider a concrete example: Nissan 15, 5751.

The Characteristics of the Jewish Year A = N + 1 = 5751

a) The rank of the year 5751 in the 19-year cycle

$[5751]_{19} = 13$; the year 5751 is the 13th year of the cycle 303 of 19 years; it is a regular year preceding a leap year.

b) The *molad* of the year 5751

The number of Jewish months preceding the *molad* of year 5751 is given by the fundamental formula of the Jewish calendar:[4]

1 *Scripta Mathematica* Vol. IX, pp. 191-196 and 274-277.

2 Gauss, Werke VI Bd. 1874, pp. 80-81. Berechnung des Judischen Osterfestes.
Zach's Monatliche Correspondenz zur beforderung der Erd und Himmelskunde, Mai 1802, p. 435.
Different authors tried to demonstrate this formula:
Demonstration des Formules de Mr. Gauss pour déterminer le jour de Pâque des Juifs, Correspondance Astronomique, Géographique, Hydrographique et Statistique du Baron De Zach, Genova, 1813.
Ableitung der gausschen formel zur bestimmung des Judischen Osterfetes, M. Hamburger, Crelles Journal für die reine und angewandte Mathematik, Band 116 (1896).
Ida Rhodes, "Computation of the dates of the Hebrew New Year and Passover," *Comp. & Maths with Appls.* Vol. 3, pp. 183-190, Pergamon Press (1977).

3 Other formulas were proposed, for example:
Eine algemeine Formel für die gesamte judischen Kalenderberechnung, Slonimsky aus Bialystock, Crelles Journal für die reine und angewandte Mathematik, Band 26 (1844).
Beitrage zur Chronologie, Nesselman in Königsberg, Crelles Journal für die reine und angewandte Mathematik, Band 28 (1844).

4 See mathematical appendix.

F_t = **INT [(235N + 1) / 19]** = INT [(235 x 5750 + 1) / 19] = 71118.

The *molad* expressed as a part of the week is:

Mol = $[31524+71118 \times 765433]_{181440}$ = $[31.524 + 71118 \times 39673]_{181440}$ = 103938 hal = 4 – 0 – 258 = (5) – 0 – 258

This *molad* is thus after 4 days and 258 *halakim* or at the beginning of the fifth day at 0h 258 halakim i.e. Wednesday at 18h 258 hal. Tishri 1 falls on Thursday.

The four gates' table gives then the *keviyah* of the year: הכז. *Rosh ha-Shanah* is a Thursday and *Pesah* is on a Saturday.

This result can also be reached directly by calculating the *molad* of the years 5751 and 5752 and the days of Tishri 1 of these two years by the application of the four rules of postponement.

F_t = INT [(235 x 5751 + 1) / 19] = 71130

Mol = $[31524 + 71130 \times 765433]_{181440}$[5] = 35694 hal = 1 – 9 – 54 = (2) – 9 – 54

Tishri 1 falls on Monday. The shift of Tishri 1 between 5751 and 5752 is thus four days and the number of days lying between these two days, exclusive of the two days of Tishri 1, is 3.[6] Therefore the year 5751 is a regular year and its length is 354 days. Thus *Rosh ha-Shanah* falls on Thursday because of the rules of the *dehiyot* and the length of the year is 354 days.

c) The year 5751 is thus a regular[7] common[8] year of 354 days beginning on a Thursday.

Nissan 15 is the 192nd day of this year and it falls on a Saturday.[9]

5 $[A]_B$ is the remainder of the division of A by B.

6 This is the algorithm described by Maimonides in *Hilkhot Kiddush ha-Hodesh* VIII, 7 and 8. He counts the number of days between the two days of Tishri 1, exclusive of the two days of Tishri 1. The length of the year is thus 353, 354 or 355 days according to whether this difference is 2, 3 or 4 for a common year; 383, 384 or 385 according to whether this difference is 4, 5 or 6 for a leap year. By contrast in his little work called: מאמר העיבור he counts the shift of *Rosh ha-Shanah* between the two years, i.e. he counts the day of *Rosh ha-Shanah* of one year + the number of days between. Therefore the length of the year is 353, 354 or 355 days according to whether the difference is 3, 4 or 5 for a common year; and 383, 384 or 385 according to whether the shift is 5, 6 or 7 for a leap year.

7 A regular year has 354 or 384 days, a defective year has 353 or 383 days and an abundant year has 355 or 385 days, according to whether the year is a common or a leap year.

8 A common year has 12 months and a leap year has 13 months.

9 See the fourteen possible calendars of the Jewish calendar: *Yesodei ha-Ibbur*, Hayim Zelig Slonimski, Warsaw 1852, end of the book. *Shearim le-Luah ha-Ivri*, Rahamim Sar Shalom, Netanya, 5744, p. 35.

The Jewish Calendar and the Julian Day

The Julian period's epoch is Monday, January 1, –4712 at noon. At this moment the number of elapsed day of the Julian period was 0. The Julian day nº 1 began on Monday at noon and ended on Tuesday at noon. Similarly, until the twentieth century, the astronomical days began at noon of the civil days of the same name.

The *molad* of *Beharad*, beginning of the Jewish era AMI, was on Sunday October 6, –3760 at 23h 204 hal; Jerusalem mean time. This moment belonged already to the second Jewish day of the week, which began at 18h, hence (2) – 5 – 204. It means the second day at 5 h and 204 *halakim*. It could be written as 1 – 2 – 204, meaning 1 day 5 h and 204 hal after the beginning of the week or 31524 hal after the beginning of the week.

Expressed in Julian days, the *molad* of *Beharad* was 347997.466203703703.

On Sunday, October 6, –3760 at noon, 347,997 days of the JP[10] had elapsed and on Monday, October 7, –3760 = Tishri 1, 1 AMI, 347,998 days of the JP had elapsed. Tishri 1, 1 AMI thus began at 347997.25 JD[11] and ended at 347998.25 JD. Tishri 1 corresponded in its majority to the day 347,998 of the JP.

There is a second style of the Jewish calendar AMII, beginning on Tishri 1, 2 AMI.

The *molad* of this year was *Weyad*: 6 – 14.

The first day of this year was Tishri 1, 1 AMII = Tishri 1, 2 AMI; it corresponds to Saturday, September 27, –3759 or 348353 JD, beginning at 348352.25 JD and ending at 348353.25 JD.

We also note that Elul 25, 1 AMI = Monday, September 22, –3759 = 348348 JD.
Elul 24, 1 AMI = Sunday, September 21, –3759 = 348347 JD.

The Year 5751 and the Civil Year

Expressed in Julian days, the *molad* of 5751 is given by the following formula:[12]

Mol= 347997.466203703 + 29.530594135804 x 71118 = 2448154.25995370370 JD

This *molad* is thus on a civil Wednesday 18h 258 hal or on a Jewish Thursday at 0h 258 hal.

Ha-Luah ve-Shimusho ba-Kronologia, A. A. Akabia, Magnes, Jerusalem 1953, pp. 50-53. These books will be named by the following abbreviations: *Yesodei*, *Shearim*, and *Kronologia*.

10 Julian period.

11 Julian day.

12 This formula gives the same result as Shram's formula.

Rosh ha-Shanah is thus Thursday, from 2448154.25 JD until 2448155.25 JD.

Tishri 1, 5751 thus corresponds to 2448155 JD and Nissan 15 = 2448155 + 191 = 2448346 JD. This day corresponds to Saturday, March 30, 1991.[13]

Indeed the beginning of that civil day corresponds to 2448345.5 JD

We add 0.5 to the JD of the beginning of the civil day: 2448346 JD.

α = INT [(2,448,346 – 1,867,216.25) / 36,524.25] = 15

A = 2,448,346 + 1 + 15 – INT (15/4) = 2,448,359

We calculate then:

B = A + 1524 = 2,449,883

C = INT [(2,449,883 – 122.1) / 365.25] = 6,707

D = INT (365.25 x 6,707) = 2,449,731

E = INT [(2,449,883 – 2,449,731) / 30.6001] = 4

The day of the month is:

B – D – INT (30.6001 x E) + F = 2,449,883 – 2,449,731 – 122 + 0.5 = 30.5[14]

The month number *m* is E – 1 = 3 = March.

The year is C – 4716 = 6707 – 4716 = 1991

The weekday is given by the JD at 0h + 1.5 = 2,448,345.5 + 1.5 = 2,448,347

$[2,448,347]_7 = 6$; it is Saturday, March 30, 1991.

Nissan 15, 5751 Expressed as a Day of the Jewish Period

If we consider that Tishri 1, 1 AMI was the first day of the Jewish period, then 1 Jewish Period = 347998 JD = Monday.

Nissan 15, 5751AMI = 2448346 – 347997 = 2100349 Jewish Period or JewP.

Nissan 15, 5751 Expressed as a Day of Kimelman's Jewish Period

In a paper published in *BDD* 16, August 2005, Arieh Kimelman proposed a Jewish era beginning on Elul 25, 1 AMI, referring to the old tradition ascribed to Rabbi

13 For the conversion of a Julian day into a civil date see Astronomical Algorithms, Jean Meeus, Willman-Bell, 1991, p. 59. This is the same reference for the determination of a weekday.

14 March 30 at noon is 30.5.

Eliezer that the first day of the creation was Sunday, Elul 25, 1 AMI.[15] However this tradition is anterior to our calendar and it does not agree with the rules of the calendar. In our calendar, Elul 25, 1 AMI is a Monday and the day to which Rabbi Eliezer referred is Elul 24, 1 AMI; this is in fact the first day of the Jewish Period of Kimelman.

The first day of his period is 348347 JD and therefore Nissan 15, 5751 corresponds to 2448346 – 348346 = 2100000 Kimelman period.

II. THE PERIOD OF THE JEWISH CALENDAR OR THE GREAT CYCLE OF THE JEWISH CALENDAR

The Remainder

The excess of a certain span of time over the greatest possible number of complete weeks is thus the remainder of the division of this span of time expressed in days by 7. Alternatively, $[\text{span}]_7$ or [span] mod 7 represents the shift of the end of this span of time in the week with regard to its beginning. If the span of time is expressed in *halakim*, then the length of the week is 181440 *halakim* and the remainder to take into consideration is the remainder of the division of the span of time by the greatest possible multiple of 181440. We will call the result of the operation $[\text{span}]_7$ or $[\text{span}]_{181440}$, the remainder.[16]

The main remainders are:

1 month			: Remainder 39673 hal
235 months	=	1 19-year cycle	: Remainder 69715 hal
3055 months	=	13 cycles	: Remainder 180535 hal = –905 hal
50760 months	=	216 cycles	: Remainder 180360 hal = –1080 hal
8527680 months	=	168 x 216 = 36288 cycles	: Remainder 0 hal
181440 months	=	772 cycles + 20 months	: Remainder 0 hal

After 216 cycles or 4104 years, the original *molad* has shifted by one hour behind.

After 168 x 216 = 36,288 cycles corresponding to 8,527,680 months and 689,472 years, the remainder is 0 and the original *molad* is again at its original position. Thus the *molad* of *Beharad* is the *molad* of the year[17] 1 AMI and it is also the *molad* of the year 689,743 AMI.

15 Vayikra Rabbah XXIX, 1.

16 Maimonides in *Hilkhot Kiddush ha-Hodesh* VI, 5 had already adopted the same terminology.

17 The *molad* of a year is the *molad* of Tishri beginning this year.

The *molad* of the year 689743 AMI is given by the formula:

347,997.466203703 + 29.530594135804 x 8,527,680 = 252,175,454.466203703 JD.[18]

The length of the Jewish period is 251,827,457 days, corresponding exactly to:

29.530594135804 x 8,527,680 = 251,827,457 days.

After 181440 months the remainder is also 0. But 181440 months corresponds to 772 cycles + 20 months. Thus after 772 cycles and 20 months, or after 14669 years and 8 months, we obtain the same *molad*.

In other words the *molad* of *Beharad* of Tishri 1 AMI was on (2) – 5 – 204.

181,440 months later the *molad* of Sivan 14670 AMI will be (2) – 5 – 204.

181,440 months later the *molad* of Tevet 29340 AMI will be (2) – 5 – 204.

And so on.

Finally, after recurring 47 times the *molad* of *Beharad* will be the *molad* of Tishri 689,743 AMI on (2) – 5 – 204.

Thus after a span of time of 181440 months the *molad* is again the same but the name of the month has changed. It is no more Tishri, but it becomes Sivan, Tevet, and so on.

We ascertain that 8,527,680 / 181440 = 47, thus each *molad* repeats itself 47 times during the Jewish period. At the 48th time we come back to the initial situation; the *molad* of *Beharad* is again a *molad* of Tishri and the second period begins again. Furthermore, we ascertain that 689,472 / 181,440 = 3.8. This is quite surprising; each *molad* would be 3.8 times the *molad* of a year during the period. This proves that the different *moladot* do not play a symmetric role with regard to the different years of the period.

If we take into consideration that (1 x 3 + 4 x 4) / (1 + 4) = 3.8 we can imagine that the 181,440 *moladot* must be divided into two groups; a first group of 36288 *moladot* would be three times the *molad* of years[19] and a second group of 145,152 *moladot* would be four times the *molad* of years.[20] In order to gain better insight into the matter we will consider a few examples showing how a *molad* reproduces during

18 At midnight the JD is 252,175,454.5 JD. One can check that this moment is indeed the beginning of Monday, November 4, 685,720 in the Gregorian calendar.

19 And 44 times the *molad* of a month different from Tishri. Note that in *B.D.D.* 22, Eran Raviv had already mentioned this fact.

20 And 43 times the *molad* of months different from Tishri.

the period. We'll first consider the *molad* (2) – 5 – 204. In the following tables, the parameters are:

* k is the order number or rank of occurrence of this *molad* during the period of the Jewish calendar before the current *molad*.
* $[F_t]$ represents the number of months before the considered *molad*.
* $[F_t]_{235}$ represents the number of months preceding the current *molad* in the current 19-year cycle.
* $[N]_{19}$ represents the number of years of the current 19-year cycle preceding the *molad* when it occurs on Tishri and is therefore the *molad* of a year.

Example 1
Molad (2) – 5 – 204. The last digit of the *molad* is 4.
The first occurrence of this *molad* is after 0 months.

Table 1: The *Molad Beharad* in the Period of the Jewish Calendar (example 1)

k	$[F_t]_{235}$	$[N]_{19}$	k	$[F_t]_{235}$	$[N]_{19}$
0	0	0	24	10	
1	20		25	30	
2	40		26	50	
3	60		27	70	
4	80		28	90	
5	100		29	110	
6	120		30	130	
7	140		31	150	
8	160	13	32	170	
9	180		33	190	
10	200		34	210	17
11	220		35	230	
12	5		36	15	
13	25		37	35	
14	45		38	55	
15	65		39	75	
16	85		40	95	
17	105		41	115	
18	125		42	135	
19	145		43	155	
20	165		44	175	
21	185	15	45	195	
22	205		46	215	
23	225		47	235	0

$[F_t]_{235}$ represents the number of months preceding the *molad* in the last 19-year cycle. As $[181{,}440]_{235} = 20$, the *molad* corresponding to k=1 is preceded by 20 months in the 19-year cycle, the *molad* corresponding to k=2 is preceded by 40 months in the last 19-year cycle, and so on.

We ascertain that the *molad* recurs 47 times in the period, but four times in the period it is the *molad* of a year, one time the *molad* of a cycle, and three times the *molad* of a year, different from the first year of a cycle. These four times are the *molad* of Tishri 1 AMI, Tishri 117358 AMI (14th year of cycle 6177), Tishri 308063 AMI (16th year of cycle 16214), and Tishri 498768 AMI (18th year of cycle 26251).

Example 2
Molad (2) – 5 – 205. The last digit of the *molad* is 5.

The first occurrence of this *molad* is after 74377 months.[21]

We ascertain that the *molad* recurs 47 times during the period of the Jewish calendar.

During this period it is 43 times the *molad* of months different from Tishri and four times the *molad* of a year: Tishri 64693 AMI (17th year of cycle 3405), Tishri 255398 AMI (19th year of cycle 13442), Tishri 446103 AMI (2nd year of cycle 23480), and Tishri 636808 (4th year of cycle 33517).

Example 3
Molad (1) – 11 – 543. The last digit of the *molad* is 3.

The first occurrence of this *molad* is after 123 months.

We ascertain that this *molad*, ending with 3, recurs 47 times in the period and is only three times the *molad* of a year during the period of the Jewish calendar: Tishri of year 11 AMI, Tishri 190716 AMI (13th year of cycle 10038), and Tishri 381421 (15th year of cycle 20075).

Example 4
Molad (4) – 17 – 928. This *molad* ends with an 8.

The first occurrence of this *molad* is after 148 months.

We ascertain that this *molad*, ending with 8, recurs 47 times during the period of the Jewish calendar but it is only three times the *molad* of a year: Tishri 13 AMI,

21 See below section V: Calculation of the First Occurrence of Any Given *Molad*.

Tishri 190718 (15th year of cycle 10038), and Tishri 498780 (11th year of cycle 26252).

In order to have a better understanding of the problem we'll examine the two first cycles of 19 years, beginning with the years 1 and 20, and examine the last digit of the *moladot*. The remainder of a 19-year cycle or 235 months is 69,715 *halakim*.

Therefore we note that the *molad* of the twentieth year, the first year of the cycle 2 is 101,239 = 31,524 + 69,715 and the *molad* of the 39th year, the first year of the 3rd cycle, is 170,954 = 101,239 + 69,715. Similarly the *molad* of any year of the cycle 2 is the *molad* of the year of the same rank in the cycle 1, increased by 69,715 *halakim*.

Table 2: The *Molad* (2) – 5 – 205 during the Period of the Jewish Calendar (example 2)

k	$[F_t]_{235}$	$[N]_{19}$	k	$[F_t]_{235}$	$[N]_{19}$
0	117		24	127	
1	137		25	147	
2	157		26	167	
3	177		27	187	
4	197	16	28	207	
5	217		29	227	
6	2		30	12	1
7	22		31	32	
8	42		32	52	
9	62		33	72	
10	82		34	92	
11	102		35	112	
12	122		36	132	
13	142		37	152	
14	162		38	172	
15	182		39	192	
16	202		40	212	
17	222	18	41	232	
18	7		42	17	
19	27		43	37	3
20	47		44	57	
21	67		45	77	
22	87		46	97	
23	107		47	117	

Table 3: The *Molad* of the Years of the First Cycle of the Jewish Period

Cycle 1			
Number of the Year	*Molad*	Number of the Year	*Molad*
1	31524	11	12423
2	144720	12	165292
3	76476	13	97048
4	47905	14	28804
5	161101	15	233
6	92857	16	113429
7	64286	17	45185
8	177482	18	16614
9	148911	19	129810
10	80667	20	101239

Table 4: The *Molad* of the Years of the Second Cycle of the Jewish Period

Cycle 2			
Number of the Year	*Molad*	Number of the Year	*Molad*
1	101239	11	82138
2	32995	12	53567
3	146191	13	166763
4	117620	14	98519
5	49376	15	69948
6	162572	16	1704
7	134001	17	114900
8	65757	18	86329
9	37186	19	18085
10	150382	20	170954

Table 5: Right Digit of the *Molad* of a Year as a Function of its Rank in the 19-year Cycle

Number of the Year	*Molad* Right Digit	Number of the Year	*Molad* Right Digit
1	4 or 9	11	3 or 8
2	0 or 5	12	2 or 7
3	6 or 1	13	8 or 3
4	5 or 0	14	4 or 9
5	1 or 6	15	3 or 8
6	7 or 2	16	9 or 4
7	6 or 1	17	5 or 0
8	2 or 7	18	4 or 9
9	1 or 6	19	0 or 5
10	7 or 2	20	9 or 4

We further note that in a 19-year cycle we encounter the last right digits 0 and 5 twice, the last right digits 2 and 7 twice, and the last right digits 1 and 6 twice. By contrast, the last right digits 4 and 9 are not equally represented: if the *molad* of the cycle ends with 4, then we have three *moladot* ending with 4 and one *molad* ending with 9. If the *molad* of the cycle ends with 9, then we have three *moladot* ending with 9 and one *molad* ending with 4. Similarly, the two last right digits 3 and 8 are not equally represented. When the *molad* of the cycle ends with 4, then we have two *moladot* ending with 3 and one *molad* ending with 8. By contrast, if the *molad* of the cycle ends with 9, then we have the *moladot* of two years ending with 8, and the *molad* of one year ending with 3.

At the scale of the complete period of the Jewish calendar representing 36,288 cycles:

* Each *molad* ending with 0 or 5 recurs 47 times during the period and among these 47 times, four times it is the *molad* of a year of rank 2, 4, 17 and 19 in the 19-year cycle.
* Each *molad* ending with 1 or 6 recurs 47 times during the period and among these 47 times, four times it is the *molad* of a year of rank 3, 5, 7 and 9 in the 19-year cycle.
* Each *molad* ending with 2 or 7 recurs 47 times during the period and among these 47 times, four times it is the *molad* of a year of rank 6, 8, 10 and 12 in the 19-year cycle.

* Each *molad* ending with 4 or 9 recurs 47 times during the period and among these 47 times, four times it is the *molad* of a year of rank 1, 14, 16 and 18 in the 19-year cycle.

It is thus one time the *molad* of a 19-year cycle, and 3 times the *molad* of a year of the cycle different from the first year of a cycle.

Each *molad* ending with 3 or 8 recurs 47 times during the period and among these 47 times, 3 times it is the *molad* of a year of rank 11, 13 and 15 in the 19-year cycle.

Thus when we consider the complete period of the Jewish calendar:

There are 181,440 possible *moladot* and thus 18,144 *moladot* ending with 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.
The *moladot* ending with 0 correspond to 18,144 x 4 = 72,576 years.
The *moladot* ending with 1 correspond to 18,144 x 4 = 72,576 years.
The *moladot* ending with 2 correspond to 18,144 x 4 = 72,576 years.
The *moladot* ending with 3 correspond to 18,144 x 3 = 54,432 years.
The *moladot* ending with 4 correspond to 18,144 x 4 = 72,576 years.
The *moladot* ending with 5 correspond to 18,144 x 4 = 72,576 years.
The *moladot* ending with 6 correspond to 18,144 x 4 = 72,576 years.
The *moladot* ending with 7 correspond to 18,144 x 4 = 72,576 years.
The *moladot* ending with 8 correspond to 18,144 x 3 = 54,432 years.
The *moladot* ending with 9 correspond to 18,144 x 4 = 72,576 years.
We can still write:

Years' *moladot* ending with 0: 72,576	Years' *moladot* ending with 5: 72,576
Years' *moladot* ending with 1: 72,576	Years' *moladot* ending with 6: 72,576
Years' *moladot* ending with 2: 72,576	Years' *moladot* ending with 7: 72,576
Years' *moladot* ending with 3: 54,432	Years' *moladot* ending with 8: 54,432
Years' *moladot* ending with 4: 72,576	Years' *moladot* ending with 9: 72,576

Total: 72,576 x 8 + 54,432 x 2 = 689,472 years.

In other words, when we consider the *moladot* of the 8,527,680 months of the Jewish period, the probability that the last digit of any *molad* is one of the figures 0, 1, 2, 3, 4, 5, 6, 7, 8 or 9 is always 10%. By contrast if we consider only the *moladot* of the 689,472 years, then the probability that the last digit of any *molad* of a year is one of the figures 0, 1, 2, 4, 5, 6, 7 or 9 is 10.53%, and the probability that the last digit of any *molad* of a year is 3 or 8 is 7.89%.

The *Dehiyot* and the Length of the Jewish Year

The length of the period of the Jewish calendar is 36,288 cycles, 689,472 years, 8,527,680 months, and 251,827,457 days.

The length of a 19-year cycle is 6939, 6940, 6941 or 6942 days.

The average length of a cycle is 6939 days + 25025 / 36288.

Now the length of 235 Jewish lunations is (29 – 12 – 793) x 235 = 6939 days + 17875/25920. It easy to check that, multiplying the numerator and the denominator of the first fraction by (7 / 5), we get the second; these two fractions are thus equal. This means that the average length of a cycle is equal to the length of 235 Jewish lunations and therefore the average length of a month is equal to the length of a Jewish lunation i.e. 29 – 12 – 793.

We ascertain that the *dehiyot* did not intervene in this reasoning. The *dehiyot* have no incidence on the average length of the Jewish calendar. Whether with *dehiyot* or without *dehiyot*, the length of the period of the Jewish calendar is the same. This fact is not generally known.[22]

III. THE FREQUENCY OF USING THE RULES OF POSTPONEMENT OR *DEHIYOT*

The problem of the quantification of the frequency of the use of rules of postponements is well known. However the solution that was previously given is an approximate solution. An exact solution requires the understanding of the generation of the *moladot* of the years of the Jewish period. The approximate solution assumes that all *moladot* of the different years of the complete period of the Jewish calendar have a uniform distribution, and therefore there is perfect proportionality between the number of years corresponding to a considered postponement, and the width of the corresponding area imposed for the *moladot* of the years.

The *keviyah* contains three letters: the first gives the weekday of *Rosh ha-Shanah*, and the third gives the weekday of the following *Pesah*. The intermediate letter indicates the length of the year. ח = defective = 353 or 383 days, כ = regular = 354 or 384 days. ש = abundant = 355 or 385 days.

22 In his paper, "Le Calendrier Juif et ses problèmes," Moïse Sibony, *Revue des Etudes Juives*, tome CXXXVI, fascicule 1-2, pp. 139-154, writes on p. 153 that the *dehiyot* have an incidence on the average length of the Jewish years and that the *dehiyot* allow for its length, thus better narrowing the length of the tropic year. Note also that Table 5 above is in fact equivalent to the table published in *B.D.D.* 22, p. 35.

Table 6: The Four Gates' Table Giving the *Keviyah* of a Year as a Function of the *Molad* of the Year

Four Gates' Table							
Common Years						Leap Years 3 – 6 – 8 – 11 – 14 – 17 – 19	
Before a Leap Year 2 – 5 – 10 – 13 – 16		After a Leap Year 1 – 4 – 9 – 12 – 15		Between Leap Years 7 – 18			
Molad	*Keviyah*	*Molad*	*Keviyah*	*Molad*	*Keviyah*	*Molad*	*Keviyah*
174960 9923	בחג	174960 9923	בחג	174960 9923	בחג	174960 22090	בחה
9924 45359	בשה	9924 42708	בשה	9924 42708	בשה	22091 45359	בשז
45360 61763	גכה	42709 61763	גכה	42709 61763	גכה	45360 71279	גכז
61764 113603	הכז	61764 113603	הכז	61764 113603	הכז	71280 90334	החא
113604 123119	השא	113604 123119	השא	113604 123119	השא	90335 123119	השג
123120 139523	זחא	123120 130007	זחא	123120 139523	זחא	123120 151690	זחג
139524 174959	זשג	130008 174959	זשג	139524 174959	זשג	151691 174959	זשה

Definition of Postponements

Table 7 provides definitions for the concept of postponements.

Table 7: Definition of Postponements

Postponement	Definition
1	*Molad Zakken*
2	*ADU*
3	*Molad Zakken + ADU*
4	*Gatrad* in a common year
5	*Betoutakfat* after a leap year
6	No postponement

The following table provides the limits of the areas of the *moladot* corresponding to each postponement. It was established from the four gates' table.

Table 8: The Postponements as a Function of the Year's Rank in a 19-Year Cycle and the *Molad* of the Year

Between Leap Years and After Leap Years			Before Leap Years			Leap Years		
1, 4, 9, 12, 15, 7 and 18			2, 5, 10, 13 and 16			3, 6, 8, 11, 14, 17 and 19		
From	Until	Postp.	From	Until	Postp.	From	Until	Postp.
9924	25919	2	9924	25919	2	9924	25919	2
25920	42708	6	25920	45359	6	25920	45359	6
42709	45359	5	45360	51839	1	45360	51839	1
45360	51839	1	51840	61763	6	51840	71279	6
51840	61763	6	61764	71279	4	71280	77759	3
61764	71279	4	71280	77759	3	77760	103679	2
71280	77759	3	77760	103679	2	103680	123119	6
77760	103679	2	103680	123119	6	123120	129599	3
103680	123119	6	123120	129599	3	129600	155519	2
123120	129599	3	129600	155519	2	155520	174959	6
129600	155519	2	155520	174959	6	174960	181439	3
155520	174959	6	174960	181439	3	181440	191363	2
174960	181439	3	181440	191363	2			
181440	191363	2						

***Dehiyah* 1**: *Molad Zakken* when the *molad* falls on Monday from noon until 5h 1079 hal p.m; thus from 45360 until 51839 included. All the years in the 19-year cycle are included. The area contains 6480 *moladot*, thus 648 for each possible right digit. We know that during the period of the Jewish calendar, each *molad* is four times the *molad* of a year except those *moladot* ending with 3 or 8, which are only three times the *molad* of a year. Therefore the total number of years with a *molad* belonging to this area is:

648 x 8 x 4 + 648 x 2 x 3 = 20736 + 3888 = 24624.[23]

23 This result is not different from the generally proposed solution considering that the number of years is proportional to the width of the area of *moladot*. In this case the proportion is 1/ 4 x 7 = 1/28 = 3.57%.

***Dehiyah* 2**: ADU. The width of this area is 3 x 4 = 12 times the width of the area of *dehiyah* 1. The width of the area is 25920 x 3 = 77760. The total number of years concerned is thus 7776 x 8 x 4 + 7776 x 2 x 3 = 24624 x 12 = 295,488 years.

***Dehiyah* 3:** *ADU* + *Molad Zakken*. The width of this area is 3 times the width of the area of *dehiyah* 1: 19440. The total number of years concerned is then:
1944 x 8 x 4 + 1944 x 2 x 3 = 24624 x 3 = 73872 years.

***Dehiyah* 4:** *Gatrad* in a common year.
Until now there was no difference between our refined analysis and the simplified method considering a perfect proportionality between the number of years in the period and the width of the areas of *moladot* corresponding to postponement under examination. But we now acknowledge that things are different.

Table 9: Counting the Number of Years Resulting from the Intervention of *Gatrad*'s Postponement for Each Rank in the 19-Year Cycle

Year's Rank in the 19-Year Cycle	Right Digit of the *Molad*	Number of Years
1	4 or 9	1904
2	0 or 5	1903
4	0 or 5	1903
5	1 or 6	1903
7	1 or 6	1903
9	1 or 6	1903
10	2 or 7	1903
12	2 or 7	1903
13	8 or 3	1903
15	3 or 8	1903
16	4 or 9	1904
18	4 or 9	1904
Total		22,839[24]

The limits of the area of the *moladot* corresponding to the postponement of *gatrad* are 61764 and 71279. The limits belong to the area, which covers 9516 *moladot*.

The total number of years resulting from the intervention of the *dehiyah* of *gatrad* is thus 22,839.

24 The common calculation is the following: [9516 / 181440] x [12/19] = 3.31%. This corresponds to 22,838.4 years in the period of the Jewish calendar.

***Dehiyah* 5:** *Betoutakfat* after a leap year.

The limits of the area of the *moladot* corresponding to the postponement of *betoutakfat* are 42709 and 45359. The limits belong to the area, which covers 9516 *moladot*.

Table 10: Counting the Number of Years Resulting from the Intervention of *Betoutakfat*'s Postponement for Each Rank in the 19-Year Cycle

Year's Rank in the 19-Year Cycle	Right Digit of the *Molad*	Number of Years
1	4 or 9	531
4	0 or 5	530
7	1 or 6	530
9	1 or 6	530
12	2 or 7	530
15	3 or 8	530
18	4 or 9	531
Total		3712[25]

The total number of years resulting from the intervention of the *dehiyah* of *betoutakfat* is thus 3712.

***Dehiyah* 6:** No postponement.
The number of years without postponement is:
689,472 – 24,624 – 295,488 – 73,872 – 22,839 – 3,712 = 268,937.

The following table summarizes our conclusions:

25 The common calculation is: [2651/ 181440] x [7/19] = 0.54%. It corresponds to 3711.4 years in the period of the Jewish calendar. See Sar Shalom, p. 37.

Table 11: Frequency of the Intervention of the Different Postponements during the Complete Period of the Jewish Calendar

Postponement	Number of Years during the Complete Period of the Jewish Calendar	Percentage or Probability of Occurrence
1	24,624	3.57
2	295,488	42.86
3	73,872	10.71
4	22,839	3.31
5	3,712	0.54
6	268,937	39.01
Total	689,472	100

IV. FREQUENCY OF THE DIFFERENT *KEVIYOT*

Let us consider in detail the case of the *keviyah*, בחה, for a leap year.

First Method

We depart from the Four Gates' table.

The area of the *moladot* corresponding to this *keviyah* is (7) – 18 to (1) – 20 – 490 included or from 174,960 to 22,090 included. It contains 28,571 *moladot*. Let us consider the different leap years as a function of their rank in the 19-year cycle.

Year 3: The right digit is 1 or 6. There are 5714 years of rank 3 in the area.
Year 6: The right digit is 7 or 2. There are 5714 years of rank 6 in the area.
Year 8: The right digit is 2 or 7. There are 5714 years of rank 8 in the area.
Year 11: The right digit is 3 or 8. There are 5714 years of rank 11 in the area.
Year 14: The right digit is 4 or 9. There are 5714 years of rank 14 in the area.
Year 17: The right digit is 5 or 0. There are 5715 years of rank 17 in the area.
Year 19: The right digit is 0 or 5. There are 5715 years of rank 19 in the area.

The total number of years whose *molad* is included in the area 174960 – 22090 is thus 40,000.[26] This gives a proportion of 40000 / 689472 = 5.8%.

26 Sar Shalom has proposed a similar reasoning in his book, p. 46. He considered an area of 28570 x (7/19) = 10525.7895 and a percentage of 10,525.7895 / 181440 = 5.8%. This calculation is approximate because the numerator is not an integer. It is even erroneous

Second Method

We depart from the 61 heads' table.

The 61 heads' table allows us to find, as a function of the areas of the *moladot* of the 19-year cycles,[27] the different types of cycles and the *keviyah* of each year. We can then find the different types of cycles including a leap year with the *keviyah* בחה. We can count the number of cycles of each type if we take into account that the right digit of the *moladot* of cycles is 4 or 9. Summing the number of cycles of each type including a leap year with the *keviyah* בחה, and summing the number of types results in the number of leap years with the *keviyah* בחה.

This second method was used in the following tables.

because the width of the area is 28571 and not 28570. I made the first approximation in my book *Hilkhot Kiddush ha-Hodesh al-pi ha-Rambam*, Jerusalem, 1996.

27 The *molad* of a cycle is the *molad* Tishri of the first year of this cycle.
Joseph Ofer has proposed in his paper entitled שכיחותן של קביעות השנה ושל ההפטרות, published in *Sinai*, Vol. 121, 1997-1998, a different solution to this problem. But again the solution is approximate.

לוח בעל ס"א ראשים

סמן	גבולת המולדות	א	ב'	ג*	ד	ה	ו*	ז	ח*	ט	י	יא*	יב	יג	יד*	טו	טז	יז*	יח	יט*	הדחיות
א	1 — 408	בחג	השא	גכז	בשה	זחא	גכז	בשה	זחג	השא	גכה	זשה	זשג	הכז	בחה	זשג	הכז	בחה	זשג	השג	ב יח
ב	1 5 333	בחג	השא	גכז	בשה	זחא	גכז	בשה	זשה	זחא	גכה	זשה	זשג	הכז	בחה	זשג	הכז	בחה	זשג	השג	ה יח
ג	1 7 870	בחג	השא	גכז	בשה	זחא	גכז	בשה	זשה	זחא	גכה	זשה	זשג	הכז	בחה	זשג	הכז	בשז	בחג	השג	ז יח
ד	1 9 204	בשה	זחא	גכז	בשה	זחא	גכז	בשה	זשה	זחא	גכה	זשה	זשג	הכז	בחה	זשג	הכז	בשז	בחג	השג	ה יח
ה	1 9 227	בשה	זחא	גכז	בשה	זשג	החא	גכה	זשה	זחא	גכה	זשה	זשג	הכז	בחה	זשג	הכז	בשז	בחג	השג	ג יח ב טו תקפט
ו	1 11 741	בשה	זחא	גכז	בשה	זשג	החא	גכה	זשה	זשג	הכז	בחה	זשג	הכז	בחה	זשג	הכז	בשז	בחג	השג	ג ט דר ז יח
ז	1 22 1051	בשה	זחא	גכז	בשה	זשג	החא	גכה	זשה	זשג	הכז	בחה	זשג	הכז	בשז	בחג	הכז	בשז	בחג	השג	ז יח
ח	1 22 1074	בשה	זחא	גכז	בשה	זשג	החא	גכה	זשה	זשג	הכז	בחה	זשג	הכז	בשז	בחג	הכז	בשז	בשה	זחג	ה יח
ט	2 — 408	בשה	זשג	החא	גכה	זשג	החא	גכה	זשה	זשג	הכז	בחה	זשג	הכז	בשז	בחג	הכז	בשז	בשה	זחג	ג יח ב טו תקפט
י	2 2 922	בשה	זשג	החא	גכה	זשג	השג	הכז	בחה	זשג	הכז	בחה	זשג	הכז	בשז	בחג	הכז	בשז	בשה	זחג	ג ט דר ז יח
יא	2 5 379	בשה	זשג	החא	גכה	זשג	השג	הכז	בחה	זשג	הכז	בחה	זשג	הכז	בשז	בחג	השא	גכז	בשה	זחג	ב יח
יב	2 14 152	בשה	זשג	החא	גכה	זשג	השג	הכז	בחה	זשג	הכז	בשז	בחג	הכז	בשז	בחג	השא	גכז	בשה	זחג	ז יח
יג	2 14 175	בשה	זשג	החא	גכה	זשג	השג	הכז	בחה	זשג	הכז	בשז	בחג	הכז	בשז	בשה	זחא	גכז	בשה	זחג	ה יח
יד	2 15 589	גכה	זשג	החא	גכה	זשג	השג	הכז	בחה	זשג	הכז	בשז	בחג	הכז	בשז	בשה	זחא	גכז	בשה	זחג	ב טו תקפט
טו	2 18 23	גכה	זשג	השג	הכז	בחג	השג	הכז	בחה	זשג	הכז	בשז	בחג	הכז	בשז	בשה	זחא	גכז	בשה	זחג	ג ט דר ז יח
טז	2 20 560	גכה	זשג	השג	הכז	בחג	השג	הכז	בחה	זשג	הכז	בשז	בחג	השא	גכז	בשה	זחא	גכז	בשה	זחג	ב יח
יז	3 1 485	גכה	זשג	השג	הכז	בחג	השג	הכז	בחה	זשג	הכז	בשז	בחג	השא	גכז	בשה	זחא	גכז	בשה	זשה	אחריה ה יח
יח	3 5 333	גכה	זשג	השג	הכז	בחג	השג	הכז	בשז	בחג	הכז	בשז	בחג	השא	גכז	בשה	זחא	גכז	בשה	זשה	ז יח
יט	3 5 356	גכה	זשג	השג	הכז	בחג	השג	הכז	בשז	בחג	הכז	בשז	בשה	זחא	גכז	בשה	זחא	גכז	בשה	זשה	ח יח
כ	3 5 379	גכה	זשג	השג	הכז	בחג	השג	הכז	בשז	בחג	הכז	בשז	בשה	זחא	גכז	בשה	זשג	החא	גכה	זשה	ג יח ב טו תקפט
כא	3 9 204	הכז	בחג	השג	הכז	בחג	השג	הכז	בשז	בחג	הכז	בשז	בשה	זחא	גכז	בשה	זשג	החא	גכה	זשה	ג ט דר ז יח
כב	3 9 227	הכז	בחג	השג	הכז	בשה	זחג	הכז	בשז	בחג	הכז	בשז	בשה	זחא	גכז	בשה	זשג	החא	גכה	זשה	ה יח
כג	3 11 741	הכז	בחג	השג	הכז	בשה	זחג	הכז	בשז	בחג	השא	גכז	בשה	זחא	גכז	בשה	זשג	החא	גכה	זשה	ב יח
כד	3 20 537	הכז	בחג	השג	הכז	בשה	זחג	הכז	בשז	בשה	זחא	גכז	בשה	זחא	גכז	בשה	זשג	החא	גכה	זשה	ה יח
כה	3 20 560	הכז	בחג	השג	הכז	בשה	זחג	הכז	בשז	בשה	זחא	גכז	בשה	זשג	החא	גכה	זשג	החא	גכה	זשה	ג יח ב טו תקפט
כו	3 22 1074	הכז	בחג	השג	הכז	בשה	זחג	הכז	בשז	בשה	זחא	גכז	בשה	זשג	החא	גכה	זשג	השג	הכז	בחה	ג ט דר ז יח
כז	4 — 408	הכז	בשה	זחג	הכז	בשה	זחג	הכז	בשז	בשה	זחא	גכז	בשה	זשג	החא	גכה	זשג	השג	הכז	בחה	ה יח
כח	4 2 922	הכז	בשה	זחג	הכז	בשה	זחג	השא	גכז	בשה	זחא	גכז	בשה	זשג	החא	גכה	זשג	השג	הכז	בחה	ב יח

	ח יח	בחה	הכז	חשג	זשג	גכה	החא	זשג	בשה	גכז	זחא	בשה	גכז	זחא	זשה	בשה	הכז	זחג	בשה	הכז	4 11 718	כט
כ טו תקפט	ג יח	בחה	הכז	השג	זשג	גכה	החא	זשג	גכה	החא	זשג	בשה	גכז	זחא	זשה	בשה	הכז	זחג	בשה	הכז	4 11 741	ל
רד ז יח	ג ט	בחה	הכז	חשג	בחג	הכז	השג	זשג	גכה	החא	זשג	בשה	גכז	זחא	זשה	בשה	הכז	זחג	בשה	הכז	4 14 175	לא
	ב יח	בחה	הכז	השג	בחג	הכז	השג	זשג	גכה	החא	זשג	בשה	גכז	זחא	זשה	גכה	השא	זחג	בשה	הכז	4 18 23	לב
ז יח	אחרית	בשז	הכז	השג	בחג	הכז	השג	זשג	גכה	החא	זשג	בשה	גכז	זחא	זשה	גכה	השא	זחג	בשה	הכז	5 1 485	לג
	ה יח	בשז	הכז	השג	בחג	הכז	השג	זשג	גכה	החא	זשג	בשה	גכז	זחא	זשה	גכה	זחא	זשה	בשה	הכז	5 2 899	לד
כ טו תקפט	ג יח	בשז	הכז	השג	בחג	הכז	השג	זשג	גכה	החא	זשג	גכה	החא	זשג	זשה	גכה	זחא	זשה	בשה	הכז	5 2 922	לה
רד	ג ט	בשז	הכז	השג	בחג	הכז	השג	בחג	הכז	השג	זשג	גכה	החא	זשג	זשה	גכה	זחא	זשה	בשה	הכז	5 5 356	לו
	ה יח	בשז	הכז	זחג	בשה	הכז	השג	בחג	הכז	השג	זשג	גכה	החא	זשג	זשה	גכה	זחא	זשה	בשה	הכז	5 5 379	לז
ז יח	ב יח	בשז	הכז	זחג	בשה	הכז	השג	בחג	הכז	השג	זשג	גכה	החא	זשג	זשה	גכה	זחא	זשה	גכה	השא	5 9 204	לח
רד ז יח	ג ט	בשז	הכז	זחג	בשה	הכז	השג	בחג	הכז	השג	זשג	גכה	החא	זשג	בחה	הכז	זשג	זשה	גכה	השא	5 9 227	לט
	ה יח	בשז	הכז	זחג	בשה	הכז	השג	בחג	הכז	השג	זשג	גכה	החא	זשג	בחה	הכז	זשג	זשה	גכה	זחא	5 18 —	מ
רד ז יח	ג ט	בשז	הכז	זחג	בשה	הכז	השג	בחג	הכז	השג	בחג	הכז	השג	זשג	בחה	הכז	זשג	זשה	גכה	זחא	5 20 537	מא
	ה יח	בשז	הכז	זחג	בשה	הכז	זחג	בשה	הכז	השג	בחג	הכז	השג	זשג	בחה	הכז	זשג	זשה	גכה	זחא	5 20 560	מב
	ב יח	גכז	השא	זחג	בשה	הכז	זחג	בשה	הכז	השג	בחג	הכז	השג	זשג	בחה	הכז	זשג	זשה	גכה	זחא	5 22 1074	מג
רד ז יח	ג ט	גכז	השא	זחג	בשה	הכז	זחג	בשה	הכז	השג	בחג	הכז	השג	זשג	בחה	הכז	זשג	בחה	הכז	זשג	6 — 408	מד
	ה יח	גכז	זחא	זשה	בשה	הכז	זחג	בשה	הכז	השג	בחג	הכז	השג	זשג	בחה	הכז	זשג	בחה	הכז	זשג	6 7 870	מה
	ז יח	גכז	זחא	זשה	בשה	הכז	זחג	בשה	הכז	השג	בחג	הכז	השג	בחג	בשז	הכז	זשג	בחה	הכז	זשג	6 11 718	מו
	ה יח	גכז	זחא	זשה	בשה	הכז	זחג	בשה	הכז	זחג	בשה	הכז	השג	בחג	בשז	הכז	זשג	בחה	הכז	זשג	6 11 741	מז
	ב יח	גכז	זחא	זשה	גכה	השא	זחג	בשה	הכז	זחג	בשה	הכז	השג	בחג	בשז	הכז	זשג	בחה	הכז	זשג	6 14 175	מח
	ה יח	גכז	זחא	זשה	גכה	זחא	זשה	בשה	הכז	זחג	בשה	הכז	השג	בחג	בשז	הכז	זשג	בחה	הכז	זשג	6 22 1051	מט
כ טו תקפ״ט	ג יח	החא	זשג	זשה	גכה	זחא	זשה	בשה	הכז	זחג	בשה	הכז	השג	בחג	בשז	הכז	בחג	בשז	הכז	זשג	6 22 1074	נ
	ז יח	החא	זשג	זשה	גכה	זחא	זשה	בשה	הכז	זחג	בשה	הכז	זחג	בשה	בשז	הכז	בחג	בשז	הכז	זשג	7 2 899	נא
	ח יח	החא	זשג	זשה	גכה	זחא	זשה	בשה	הכז	זחג	בשה	הכז	זחג	בשה	בשז	הכז	בחג	בשז	הכז	זשג	7 2 922	נב
	ב יח	החא	זשג	זשה	גכה	זחא	זשה	גכה	השא	זחג	בשה	הכז	זחג	בשה	בשז	הכז	בחג	בשז	הכז	זשג	7 5 356	נג
רד ז יח	ג ט	החא	זשג	בחה	הכז	זשג	זשה	גכה	השא	זחג	בשה	הכז	זחג	בשה	בשז	הכז	בחג	בשז	הכז	זשג	7 5 379	נד
	ב יח	החא	זשג	בחה	הכז	זשג	זשה	גכה	השא	זחג	בשה	הכז	זחג	בשה	גכז	השא	בחג	בשז	הכז	זשג	7 9 227	נה
	ה יח	החא	זשג	בחה	הכז	זשג	זשה	גכה	זחא	זשה	בשה	הכז	זחג	בשה	גכז	השא	בחג	בשז	הכז	זשג	7 14 152	נו
כ ט רד	אחריה	השג	זשג	בחה	הכז	זשג	זשה	גכה	זחא	זשה	בשה	הכז	זחג	בשה	גכז	השא	בחג	בשז	הכז	בחג	7 16 689	נז
	ז יח	השג	זשג	בחה	הכז	זשג	זשה	גכה	זחא	זשה	בשה	הכז	זחג	בשה	גכז	זחא	בשה	בשז	הכז	בחג	7 18 —	נח
	ה יח	השג	זשג	בחה	הכז	זשג	זשה	גכה	זחא	זשה	גכה	השא	זחג	בשה	גכז	זחא	בשה	בשז	הכז	בחג	7 18 23	נט
	ב יח	השג	זשג	בחה	הכז	זשג	זשה	גכה	זחא	זשה	גכה	השא	זחג	בשה	גכז	זחא	בשה	בשז	הכז	בחג	7 20 537	ס
רד ז יח	ג ט	השג	זשג	בחה	הכז	זשג	בחה	הכז	זשג	זשה	גכה	השא	זחג	בשה	גכז	זחא	בשה	בשז	הכז	בחג	7 28 560	סא

Table 12: Number of 19-Year Cycles Included in Each Head

Rank of the Head	From	Until	Number of Cycles	Rank of the Head	From	Until	Number of Cycles
1	408	5732	1065	32	97223	105244	1605
2	5733	8429	540	33	105245	106738	298
3	8430	9923	298	34	106739	106761	5
4	9924	9946	5	35	106762	109435	535
5	9947	12620	535	36	109436	109458	4
6	12621	24810	2438	37	109459	113603	829
7	24811	24833	4	38	113604	113626	5
8	24834	26327	299	39	113627	123119	1899
9	26328	29001	535	40	123120	125816	539
10	29002	31698	539	41	125817	125839	5
11	31699	41191	1899	42	125840	128513	534
12	41192	41214	5	43	128514	130007	299
13	41215	42708	298	44	130008	138029	1605
14	42709	45382	535	45	138030	142197	833
15	45383	48079	540	46	142198	142220	5
16	48080	53404	1065	47	142221	144894	535
17	53405	57572	833	48	144895	154410	1903
18	57573	57595	5	49	154411	154433	4
19	57596	57618	4	50	154434	158578	829
20	57619	61763	829	51	158579	158601	5
21	61764	61786	5	52	158602	161275	535
22	61787	64460	535	53	161276	161298	4
23	64461	73976	1903	54	161299	165466	834
24	73977	73999	5	55	165467	170791	1065
25	74000	76673	534	56	170792	173488	539
26	76674	78167	299	57	173489	174959	295
27	78168	80841	535	58	174960	174982	4
28	80842	90357	1903	59	174983	177656	535
29	90358	90380	5	60	177657	177679	5
30	90381	93054	535	61	177680	407	833
31	93055	97222	833	**Total**			36288

Table 13: Number of Common Years בחג in the Period of the Jewish Calendar

Rank of the Years in the 19-Year Cycle	Rank Number of the Heads of the Cycle	Number of Years
1	From 58 until 61 and from 1 till 3 included	3280
2	From 21 until 26 included	3281
4	From 51 until 58 included	3281
5	From 15 until 21 included	3281
7	From 46 until 51 included	3281
9	From 18 until 23 included	3281
10	From 41 until 46 included	3281
12	From 12 until 18 included	3281
13	From 36 until 41 included	3281
15	From 7 until 12 included	3281
16	From 31 until 36 included	3280
18	From 3 until 7 included	3280
	Total	39369

Table 14: Number of Common Years בשה in the Period of the Jewish Calendar

Rank of the Years in the 19-Year Cycle	Rank Number of the Heads of the Cycle	Number of Years
1	From 4 until 13 included	6557
2	From 71 until 37 included	7087
4	From 59 until 61 and from 1 until 8 included	6557
5	From 21 until 31 included	7087
7	From 52 until 61 and from 1 until 4 included	6557
9	From 24 until 34 included	6557
10	From 47 until 59 included	7087
12	From 19 until 29 included	6557
13	From 42 until 52 included	7087
15	From 13 until 24 included	6557
16	From 37 until 47 included	7088
18	From 8 until 19 included	6557
	Total	81335

The number of each of the other twelve types of calendar years in the Jewish calendar period can be calculated in the same manner.

Table 15: Distribution of the 689,472 Years of the Jewish Period as a Function of Their *Keviyah*

Common Years			Leap Years		
Keviyah	Number of Years	Percentage or Probability	*Keviyah*	Number of Years	Percentage or Probability
בחג	39369	5.7100	בחה	40000	5.8015
בשה	81335	11.7967	בשז	32576	4.7248
גכה	43081	6.2484	גכז	36288	5.2632
הכז	124416	18.0451	החא	26677	3.8692
השא	22839	3.3125	השג	45899	6.6571
זחא	29853	4.3298	זחג	40000	5.8015
זשג	94563	13.7153	זשה	32576	4.7248
Total	435456			254016	

Table 16: Distribution of the 689,472 Years of the Jewish Period as a Function of Their Length

Year's Length	Number of Years	Percentage or Probability
353	69222	10.0399
354	167497	24.2935
355	198737	28.8245
383	106677	15.4723
384	36288	5.2632
385	111051	16.1067
	689472	100
Length of the period	251,827,457 days	

Table 17: Distribution of the 689,472 Years of the Jewish Period as a Function of the Weekday of the First Day of *Rosh ha-Shanah*

Rosh ha-Shanah	Number of Years	Percentage or Probability	Preceding *Pesah*
Thursday	219831[28]	31.8840	Tuesday
Saturday	196992[29]	28.5714	Thursday
Monday	193280[30]	28.0330	Saturday
Tuesday	79369[31]	11.5116	Sunday

Table 18: Distribution of the 689,472 Years of the Jewish Period as a Function of the Weekday of the First Day of *Pesah* Preceding *Rosh ha-Shanah*

Pesah	Number of Years	Percentage or Probability	Preceding *Purim*	*Tisha be-Av*
Tuesday	219831	31.8840	Sunday	Tuesday
Thursday	196992	28.5714	Tuesday	Thursday
Saturday	193280	28.0330	Thursday	Saturday
Sunday	79369	11.5116	Friday	Sunday

V. CALCULATION OF THE FIRST OCCURRENCE OF ANY GIVEN *MOLAD*[32]

Calculation of the first occurrence of the *molad* (2) – 5 – 205 = 31525

This *molad* differs from *Beharad* by 1 hal.

The length of a month is 765,433 hal and $[765{,}433]_{181440} = 39673$.

28 39369 + 94563 + 45899 + 40000 = 219831

29 81335 + 43081 + 40000 + 32576 = 196992

30 124416 + 32576 + 36288 = 193280

31 22839 + 29853 + 26677 = 79369

32 This solution was proposed in my book *Hilkhot Kiddush ha-Hodesh al-pi ha-Rambam*, Jerusalem, 1996. At the same time, the late Hanokh Merzbach, nephew of R' Yona Merzbach, proposed a similar solution in his paper entitled חישובי מולדות על ידי נוסחאות, *Sinai*, Vol. 117, pp. 284-286. The method is slightly different; it makes use of a few steps and seems longer. It has, however, the advantage of manipulating smaller numbers and therefore it requires a lower-performing calculator (fewer working digits). However, the solution of Hanokh Merzbach was presented as a formula without explaining how the different coefficients were found.

If we consider that this *molad* is the *molad* of the month X+1 and is preceded by X months, we can then write: $[X \times 39673]_{181440} = 1$.

We demonstrate[33] that the number satisfying this relation is the number **X = 74377**. One ascertains that 74377 is a prime number.
Thus after 74377 months we obtain the *molad Beharad* increased by 1 hal.

Calculation of the first occurrence of the *molad Beharad* increased by T hal
We know that after 74377 months the *molad* is the *molad Beharad* + 1, thus after A x 74377 or already after $[A \times 74377]_{181440}$ months the *molad* will then be *Beharad* + A.

Thus $[A \times 74{,}377 \times 39{,}673]_{181440} = A$

Now we know that after a month the *molad* increases by 39673 hal, and after A months, the *molad* increases by T = A x 39673 hal.

Thus $A = [(A \times 39{,}673) \times 74{,}377]_{181440}$

And $\mathbf{A = [T \times 74{,}377]_{181440}}$ where T is the increase of the *molad* after A months.
If we choose any *molad*, the increase of the *molad* is T = *molad* – 31,524. It happens after A months. A is given by A = [T x 74,377] + k x 181440.

Consequences
When the rank of a month increases by 1, then the *molad* increases by 39673.
When the rank of a month increases by K, then the *molad* increases by K x 39673 and $K = [K \times 39673 \times 74377]_{181440}$ where K is an integer varying from 1 until 181440.
When K = 181440 then all the *moladot* have worked a first time.

When the *molad* increases by 1, then the rank of the month increases by 74377.

When the *molad* increases by T, then the rank of the month increases with $A = [T \times 74377]_{181440}$.

Calculation of the other 46 occurrences of any *molad*
We have seen in section II (examples 1 to 4) how we can build the table of the 46 following occurrences of a given *molad* once we know the first occurrence of this *molad*.[34]

33 See mathematical appendix.
34 See Tables 1 and 2.

Example 1

When does the *molad* (1) – 17 – 107 = 18467[35] occur for the first time? When will this *molad* be the *molad* of Nissan as Maimonides considered it?

T = 18,467 – 31,524 + 181,440 = 168,383

A = $[168{,}383 \times 74377]_{181440}$ = 107,831 = 458 x 235 + 201 months.

This *molad* occurs after 458 cycles and 201 months, and it is the *molad* of Shevat 8719.

181440 months later the same *molad* recurs: A= 107,831 + 181,440 = 289,271 = 1230 x 235 + 221. This is the *molad* of Elul 23388.

181440 months later the same *molad* recurs: A = 289,271 + 181,440 = 470,711 = 2003 x 235 + 6. This is the *molad* of Nissan 38058. This is the month of Nissan to which Maimonides referred.

Example 2

When does the *molad* (1) – 11 – 543 or 12423 occur for the first time?

T = 12423 – 31524 = –19101 = 162339

A = $[T \times 74377]_{181440}$ = 123; thus after 123 months

Example 3

When does the *molad* (4) – 17 – 928 = 97048 occur for the first time?

T = 97048 – 31524 = 65524

A = $[T \times 77377]_{181440}$ = 148; thus after 148 months.

Example 4

Following is a demonstration of the method proposed by Hanokh Merzbach.[36]

a) When does the *molad* (7) – 0 – 0 occur?

This *molad* is 155520 hal.

T = 155520 – 31524 = 123996.

A= $[123996 \times 74377]_{181440}$ = 36732.

This *molad* occurs after 36732 months.

b) How many months afterwards does the *molad* (1) – 0 – 0 occur?

This *molad* is 181440 hal.

T = 181440 – 31524 = 149916.

A = $[149916 \times 74377]_{181440}$ = 88572 = 36732 + 51840.

35 See *Hilkhot Kiddush ha-Hodesh* VI : 7.

36 *Sinai*, Vol 117, pp. 284-286.

This *molad* occurs 88572 months after *Beharad*, and 51840 months after the *molad* (7) – 0 – 0.

c) How many months afterwards does the *molad* (1) – 1 – 0 occur?
This *molad* is 182520 hal.
T = 182520 – 31524 = 150996.
A = $[150996 \times 74377]_{181440}$ = 37812 = $[88572 + 130680]_{181440}$.
This *molad* occurs 37812 months after *Beharad* and 130680 months after the *molad* (1) – 0 – 0.

d) How many months afterwards does the *molad* (1) – 1 – 1 occur?
According to the theory developed above, we know that this *molad* occurs 74377 months after the *molad* (1) – 1 – 0.

e) After how many months does the *molad* (D) – H – hal occur?

First solution
Mol = 25920 x (D – 1) + 1080 x H + hal.
T = Mol – 31524.
A = $[T \times 74377]_{181440}$.

Second solution (Hanokh Merzbach)
A = $[36732 + 51840 \times D + 130680 \times H + 74377 \times hal]_{181440}$.

VI. CALCULATION OF THE CASES WHEN ANY GIVEN *MOLAD* IS THE *MOLAD* OF A YEAR

We know that any *molad* recurs 47 times during the period of the Jewish calendar and among these 47 times, it is four times the *molad* of a year, except when the right digit of this *molad* is 3 or 8; then the *molad* recurs also 47 times during the period of the Jewish calendar, but it is only the *molad* of a year for three times.

The problem is to see if it is possible to find a direct solution to this problem without the construction of the complete table of the 47 occurrences of the given *molad*. In order to solve this problem, we refer to Table 5 showing the *molad* of the years of the first 19-year cycle. We have already noted that when we progress from a year of rank *r* of the first cycle to the year of the same rank of the second cycle, the *molad* of this year is increased by 69715 and the right digit of the *molad* increases by 5. We also know that the right digit depends on the year's rank in the cycle.

1. Given any year of rank r of the cycle 1, we calculate the number y of cycles after which the *molad* of the year of the same rank r increased with 5.[37]

When we progress from a year of rank r in cycle 1 to cycle 2, or from any cycle to the next, the *molad* increases by 69715.

When we progress from cycle 1 to cycle $1 + y$, the *molad* increases by 69715 x y.

We must thus find an integer y which satisfies the equation: $[y \times 69715]_{181440} = 5$.

Or after simplification: $\mathbf{[y \times 13943]_{36288} = 1}$.

We demonstrate[38] that the number satisfying this relation is the number y = **6215**. Thus after 6215 cycles, the *molad* of the year of rank r increased by 5.

Verification

The *molad* of the year of rank $r = 1$ of the cycle 1 is (2) – 5 – 204 = 31524.

The *molad* of the year of rank $r = 1$ of the cycle 6216 is 31529. Indeed:

Mol = 31524 + $[6215 \times 69715]_{181440}$ = 31529.

We can also write: Mol = 31524 + $[6215 \times 235 \times 39673]_{181440}$ = 31529.

2. Given any year of rank r of cycle 1, calculate the number y of cycles, after which the *molad* of the year of the same rank r increased with U hal.[39]

After 6215 cycles, the *molad* of the year of rank r increased by 5.

After B x 6215 cycles, the *molad* of the year of rank r increased by 5 x B.

Thus $[B \times 6215 \times 69715]_{181440} = 5 \times B$.

But after 1 cycle, the *molad* of the year of rank r increases by 69715 and after B cycles, the *molad* of the year of rank r increases by U = B x 69715. Thus:

$5 \times B = [(B \times 69715) \times 6215]_{181440} = [U \times 6215]_{181440}$. U is the increase of the *molad* of the year of rank r, i.e. the difference between the given *molad* and the *molad* of the year of rank r in cycle 1. The formula allows one to calculate B, the number of cycles after which the *molad* increases by U.

The formula is thus $5 \times B = [U \times 6215]_{181440}$

One can also write $\mathbf{B = [U \times 1243]_{36288}}$

37 The problem has a similitude with the problem considered at the beginning of section V: Calculation of the First Occurrence of Any Given *Molad*. Note that an increase of the *molad* by 5 is the smallest possible increase.

38 See mathematical appendix.

39 B is of course a multiple of 5.

Example

Let us consider the *molad* (2) – 5 – 205 = 31525.

We examined this *molad* at the beginning of this paper and established Table 2, which shows all 47 occurrences of this *molad* during the period of the Jewish calendar.

We observed that this *molad* is the *molad* of a year after a certain number of months:

74377 + 30 x 181440 = 5,517,577 months = 23479 cycles + 1 year
74377 + 43 x 181440 = 7,876,297 months = 33516 cycles + 3 years
74377 + 4 x 181440 = 800,137 months = 3404 cycles + 16 years
74377 + 17 x 181440 = 3,158,857 months = 13441 cycles + 18 years

We refer to Table 3 of the first cycle.

The years of reference are those years which have a right digit of 5 or 0.

Years of Reference

Year of rank 2: Mol= 144720. U = 31525 – 144720 = –113195 = 31957[40]
Year of rank 4: Mol = 47905. U = 31525 – 47905 = –16380 = 19908
Year of rank 17: Mol = 45185. U= 31525 – 45185 = –13660 = 22628
Year of rank 19: Mol = 129810. U = 31525 – 129810 = –98285 = 10579
We can now generate the researched *moladot*:

Year of Rank 2

B= $[1243 \times 31957]_{36288}$ = 23479. It is the *molad* Tishri of the 2nd year of the cycle 23480.

Year of Rank 4

B= $[1243 \times 19908]_{36288}$ = 33516. It is the *molad* Tishri of the 4th year of the cycle 33517.

Year of Rank 17

B= $[1243 \times 22628]_{36288}$ = 3404. It is the *molad* Tishri of the 17th year of the cycle 3405.

40 After addition of a multiple of 36288.

Year of Rank 19
B= $[1243 \times 10579]_{36288}$ = 13441. It is the *molad* of Tishri of the 19th year of the cycle 13442.

VII. MATHEMATICAL APPENDIX

The Fundamental Formula of the Jewish Calendar

The number of months preceding the *molad* of the Jewish year N + 1 is given by $F_t = INT [(235N + 1) / 19]$[41]

The following table illustrates the practical demonstration of this formula:

N	F_t	N	F_t	N	F_t	N	F_t
1	12	6	74	11	136	16	197
2	24	7	86	12	148	17	210
3	37	8	99	13	160	18	222
4	49	9	111	14	173	19	235
5	61	10	123	15	185	20	247

The numbers of the columns F_t are indeed the number of months preceding the beginning of the different years of the 19-year cycle.

This formula is general. It allows one to calculate the *molad* of any year.

Calculation of the First Occurrence of the *Molad* (2) – 5 – 205 = 31525

This *molad* differs from *Beharad* by 1 hal.
The length of a month is 765,433 hal and $[765{,}433]_{181440} = 39673$.
If we consider that this *molad* is the *molad* of the month X+1 and is preceded by X months.

1) We can then write: $[X \times 39673]_{181440} = 1$
Solution of the equation: $[X \times 39673]_{181440} = 1$

We note that:
2) $181440 = (2)^6 \times (3)^4 \times 5 \times 7 = (72)^2 \times 5 \times 7$
3) $[39673]_9 = [39673]_{24} = [39673]_{72} = 1$
4) $[2 \times 39673]_5 = [2 \times 39673]_7 = 1$
5) $[a \times b]_n = [[a]_n \times [b]_n]_n$

41 This formula was given for the first time in *Al ha-Sheminit*, Y. Loewinger, Tel Aviv, 1986.

6) $[a + b]_n = [[a]_n + [b]_n]_n$

7) Equation (1) can be written in the following way: $[X \text{ x } 39673 - 1]_{181440} = 0$

It involves, because of equation (2), the following equations:

8) $[X \text{ x } 39673 - 1]_5 = 0$

9) $[X \text{ x } 39673 - 1]_{5 \text{ x } 7} = 0$

10) $[X \text{ x } 39673 - 1]_{5 \text{ x } 7 \text{ x } 72} = 0$

11) $[X \text{ x } 39673 - 1]_{5 \text{ x } 7 \text{ x } 72 \text{ x } 72} = 0$

Equation (8) gives:

$[2 \text{ x } X \text{ x } 39673 - 2]_5 = 0$

Because of (4) and (5):

12) $[X - 2]_5 = 0$ and $X = 2 + k_1 \text{ x } 5$

Equation (9) gives:

$[X \text{ x } 39673 - 1]_{35} = 0$

$[(2 + k_1 \text{ x } 5) \text{ x } 39673 - 1]_{35} = 0$

13) $[2 \text{ x } 39673 - 1 + 5 \text{ x } k_1 \text{ x } 39673]_{35} = 0$

$2 \text{ x } 39673 - 1 = 15369 \text{ x } 5 = 2267 \text{ x } 5 \text{ x } 7$

Thus equation (13) becomes: $[5 \text{ x } k_1 \text{ x } 39673]_{35} = 0$

$[k_1 \text{ x } 39673]_7 = 0$

But $[39673]_7 = 4$ and therefore $[k_1]_7 = 0$ and $k_1 = 7xk_2$

14) Therefore $X = 2 + 5 \text{ x } 7 \text{ x } k_2$

Equation (10) gives:

$[X \text{ x } 39673 - 1]_{5 \text{ x } 7 \text{ x } 72} = 0$

Introducing (14) we get: $[39673 \text{ x } (2 + 5 \text{ x } 7 \text{ x } k_2) - 1]_{5 \text{ x } 7 \text{ x } 72} = 0$ or

15) $[79346 + 39673 \text{ x } 5 \text{ x } 7 \text{ x } k_2 - 1]_{5 \text{ x } 7 \text{ x } 72} = 0$

$79345 = 2267 \text{ x } 5 \text{ x } 7$ and therefore (15) becomes

$[2267 + 39673 \text{ x } k_2]_{72} = 0$. Now we note that $[2267]_{72} = 35$

Thus $[39673 \text{ x } k_2]_{72} = -35 = 37$. Now we note that $[39673]_{72} = 1$

Thus $[k_2]_{72} = 37$ and therefore $k_2 = 37 + 72 \text{ x } k_3$

16) Equation (15) gives $X = 2 + 5 \text{ x } 7 \text{ x } (37 + 72 \text{ x } k_3)$

$X = 1297 + 5 \text{ x } 7 \text{ x } 72 \text{ x } k_3$

Equation (11) gives:

$[X \text{ x } 39673 - 1]_{5 \text{ x } 7 \text{ x } 72 \text{ x } 72} = 0$

Introducing (16) we get: $[39673 \text{ x } (1297 + 5 \text{ x } 7 \text{ x } 72 \text{ x } k_3) - 1]_{5 \text{ x } 7 \text{ x } 72 \text{ x } 72} = 0$ or

17) $[51{,}455{,}880 + 5 \text{ x } 7 \text{ x } 72 \text{ x } 72 \text{ x } k_3]_{5 \text{ x } 7 \text{ x } 72 \text{ x } 72} = 0$

Now 51,455,880 = 20419 x 5 x 7 x 72 and therefore (17) becomes:
$[20419 + k_3]_{72} = 0$
Therefore $k_3 = [-20419]_{72} \times k_4 = 29 + 72 \times k_4$
Equation (16) becomes $X = 1297 + 5 \times 7 \times 72\ (29 + 72 \times k_4)$
$X = 1297 + 73080 + 181440\ k_4$

If we replace k_4 by k then the solution to our problem is:
X = 74377 + k x 181440

The *molad* (2) – 5 – 205 occurs for the first time after 74377 months and then it recurs 46 other times after the 46 multiples of 181440 months.

Thus any *molad* is increased by 1 after 74377 months.

Alternative Solution
The problem can be posed in the form of a linear Diophantine equation. The equation (1) is equivalent to the linear Diophantine equation: X x 39673 – t x 181440 = 1

We develop Euclid's algorithm:

181440	=	4	x	39673	+	22748
39673	=	1	x	22748	+	16925
22748	=	1	x	16925	+	5823
16925	=	2	x	5823	+	5279
5823	=	1	x	5279	+	544
5279	=	9	x	544	+	383
544	=	1	x	383	+	161
383	=	2	x	161	+	61
161	=	2	x	61	+	39
61	=	1	x	39	+	22
39	=	1	x	22	+	17
22	=	1	x	17	+	5
17	=	3	x	5	+	2
5	=	2	x	2	+	1
2	=	2	x	1	+	0

Then we start with the last equation and we inductively back-substitute, simplifying at each step:

$1 = 5 - 2 \times 2 = 5 - 2\,(17 - 3 \times 5) = -2 \times 17 + 7 \times (22 - 17)$
$= 7 \times 22 - 9 \times (39 - 22) = -9 \times 39 + 16 \times (61 - 39) = 16 \times 61 - 25 \times (161 - 61 \times 2)$
$= -25 \times 161 + 66 \times (383 - 161 \times 2) = 66 \times 383 - 157 \times (544 - 383)$
$= -157 \times 544 + 223 \times (5279 - 544 \times 9) = 223 \times 5279 - 2164 \times (5823 - 55279)$
$= -2164 \times 5823 + 2387 \times (16925 - 5823 \times 2) = -6938 \times 22748 + 9325 \times (39673 - 22748)$
$= 9325 \times 39673 - 16263 \times (181440 - 4 \times 39673) = 74377 \times 39673 - 16263 \times 181440$.

Thus: **74377 x 39673 – 16263 x 181440 = 1.**

The general solution of the linear Diophantine equation is given by:
$X = 74377 + k \times 181440$
$t = 16263 + k \times 39673$

Solution of the Equation: $[y \times 13943]_{36288} = 1$

y must be an integer.

18) We note that 36288 = 6 x 7 x 8 x 9 x 12
19) $[y \times 13943]_{36288} = 1$

Equation (19) can be written in the following way:

20) $[y \times 13943 - 1]_{36288} = 0$

Because of equation (18) it involves the following equations:

21) $[y \times 13943 - 1]_{7} = 0$
22) $[y \times 13943 - 1]_{6 \times 7} = 0$
23) $[y \times 13943 - 1]_{6 \times 7 \times 8} = 0$
24) $[y \times 13943 - 1]_{6 \times 7 \times 8 \times 9} = 0$
25) $[y \times 13943 - 1]_{6 \times 7 \times 8 \times 9 \times 12} = 0$

Equation (21) gives:

26) $[6y \times 13943 - 6]_{7} = 0$
27) But $[6 \times 13943]_{7} = 1$ therefore (26) becomes $[y - 6]_{7} = 0$
28) From (27) we deduce: $\mathbf{y = 6 + k_1 \times 7}$

When we introduce (28), equation (22) now gives:
$[(6 + \mathbf{k_1} \times 7) \times 13943 - 1]_{6 \times 7} = 0$
$[83658 + 7k_1 \times 13943 - 1]_{6 \times 7} = 0$
We note that 83657 = 7 x 11951
Hence: $[11951 + k \times 13943]_{6} = 0$
We note that $[11951]_{6} = 5$ and $[13943]_{6} = 5$
Therefore $[5 + k_1 \times 5]_{6} = 0$

$[5 - k_1]_6 = 0$ and $[k_1]_6 = 5$

29) Finally $k_1 = 5 + 6\,k_2$

Introducing (29) into equation (28) we obtain:

30) $y = 6 + 7 \text{ x } (5 + 6k_2)$ or $y = 41 + 42\,k_2$

Equation (23) gives:

$[y \text{ x } 13943 - 1]_{6\text{x}7\text{x}8} = 0$. We introduce (30):

$[(41 + 42k_2) \text{ x } 13943 - 1]_{6\text{x}7\text{x}8} = 0$

$[571663 + 42k_2 \text{ x } 13943 - 1]_{6\text{x}7\text{x}8} = 0$

Now $571662 = 42 \text{ x } 13611$, therefore

$[13611 + k_2 \text{ x } 13943]_8 = 0$

Now $[13611]_8 = 3$ and $[13943]_8 = 7$

$[3 + k_2 \text{ x } 7]_8 = 0$ or $[3 - k_2]_8 = 0$ and finally $k_2 = 3 + 8 \text{ x } k_3$

Equation (30) becomes $y = 41 + 42 \text{ x } (3 + 8 \text{ x } k_3)$; or

31) $y = 167 + 336k_3$

Equation (24) gives:

$[y \text{ x } 13943 - 1]_{6\text{x}7\text{x}8\text{x}9} = 0$. We introduce (31):

32) $[(167 + 336 \text{ x } k_3) \text{ x } 13943 - 1]_{6\text{x}7\text{x}8\text{x}9} = 0$

We note that $(167 \text{ x } 13943 - 1) = 6 \text{ x } 7 \text{ x } 8 \text{ x } 6930 = 336 \text{ x } 6930$, therefore (32) becomes:

$[6930 + k_3 \text{ x } 13943]_9 = 0$

$[k_3 \text{ x } 13943]_9 = 0$ because $[6930]_9 = 0$. Now $[13943]_9 = 2$, thus

33) $[2k_3]_9 = 0$ and therefore $2k_3 = 0 + 9k_4 = 9\,k_4$

Introducing (33) into (31) we get:

34) $y = 167 + 1512k_4$

Equation (25) gives:

$[y \text{ x } 13943 - 1] = 0$. Introducing (34) we get

35) $[(167 + 1512k_4) \text{ x } 13943 - 1]_{6\text{x}7\text{x}8\text{x}9\text{x}12} = 0$

We note that $(167 \text{ x } 13943 - 1) = 7 \text{ x } 8 \text{ x } 9 \text{ x } 4620$

and $1512 = 7 \text{ x } 8 \text{ x } 9 \text{ x } 3$; therefore (35) becomes

36) $[4620 + 41829k_4]_{6\text{x}12} = 0$

But $[4620]_{6\text{x}12} = 12$ and $[41829]_{6\text{x}12} = 69$, therefore (36) becomes:

$[12 + 69k_4]_{6\text{x}12} = 0$

or: $[12 - 3k_4]_{6\text{x}12} = 0$ thus

37) $3k_4 = 12 + 72k_5$.

Introducing (37) into (34) we obtain:

$y = 167 + 504 \times (12 + 72\ k_5) = 167 + 6048 + 36288k_4$

Hence the solution:

$y = 6215 + k \times 36288$

Alternative Solution

The problem can be posed in the form of a linear Diophantine equation. The equation (1) is equivalent to the linear Diophantine equation: $y \times 13943 - t \times 36288 = 1$.

We develop Euclid's algorithm:

36288 = 2 x 13943 + 8402
13943 = 1 x 8402 + 5541
8402 = 1 x 5541 + 2861
5541 = 1 x 2861 + 2680
2861 = 1 x 2680 + 181
2680 = 14 x 181 + 146
181 = 1 x 146 + 35
146 = 4 x 35 + 6
35 = 5 x 6 + 5
6 = 1 x 5 + 1
5 = 5 x 1 + 0

Then we start with the last equation and we inductively back-substitute, simplifying at each step:

1 = 6 – 5 = 6 – (35 – 6 x 5) = –35 + 6 x (146 – 35 x 4) = 6 x 146 – 25 x (181 – 146)
= –25 x 181 + 31 x (2680 – 181 x 14) = 31 x 2680 – 459 x (2861 – 2680)
= –459 x 2861 + 490 x (5541 – 2861) = 490 x 5541– 949 x (8402 – 5541)
= –949 x 8402 + 1439 x (13943 – 8402) = 1439 x 13943 – 2388 x (36288 – 2 x 13943) = 6215 x 13943 – 2388 x 36288.

Thus: 6215 x 13943 – 2388 x 36288 = 1

The general solution of the linear Diophantine equation is given by:

$y = 6215 + k \times 36288$

$t = 2388 + k \times 13943$

VIII. HISTORICAL APPENDIX

I had the occasion[42] to consult the manuscript of the Zentralbibliothek Zürich: Ms Heid. 180. This manuscript corresponds to the first part of Raphael Hanover's *Luhot ha-Ibbur*. It is identical to the printed text, with exactly the same title page and remarks as printed on the back of the title page.

However, tables 1 to 7, essential for the understanding of the book, are missing.

At the end of the book, after its formal end, we find a fifth example and finally an additional note which seems sufficiently important to be published here. From these additions we can understand that this manuscript was written by a pupil from a first manuscript (probably the autograph manuscript) in 1752.[43] We note that in the title page of the present manuscript the author is qualified as:[44]

המהנדס התוכן ופילוסוף **הגדול**, **רבן של כל בני הגולה**, החבר רבי רפאל הלוי, **נרו יאיר ויזרח**, מהאנובר.

On page 42 of the manuscript, we find the following text, which is essential for the history of the Jewish calendar and directly related to the present paper:

גם זה למדתי מפי גדול, הלה הוא התוכן מ"ה ר' רפאל נר"ו.

אם יאמר לך אדם, המולד היה או יהיה באותו יום ובאותה שעה ובאותם רגעים.

תגרע מהמולד הניתן לך 2-5-204 ומהמספר הנשאר תעשה שעות והשעות לחלקי תתר"ף ואז כשיהיה לך יחד מכל הימים והשעות חלקי תתר"ף תכפול אותן על ידי 74377 והיוצא תחלוק ע"י 181440. ואחר כל החילוק האחרון שאז יתחלק פחות מן 181440 אותו המספר הוא מספר של החודש שאתה עומד בו,[45] דהיינו הניתן לך בראשונה. ותעשה בזה האופן. תחלוק אותן המותרות שנשאר בידך על ידי 235 שהוא חודשי המחזור קטן עם ז' עיבורים שעולין רל"ה חדשים והיוצאין יהיו מחזורים והנשארין פחות מן רל"ה יהיו חדשים ותחלוק אותן חדשים ע"י 12 והיוצאין יהיו שנים ותוסיף על השנים האלו העיבורין שעברו והנשארים יהיו חדשים שעברו. ואותו מולד הניתן לך הוא המולד מחדש שאתה עומד בו. ועתה, אתה המעיין ראה והבן את אשר לפניך המשל הזה.

למשל ניתן לך המולד היה או יהיה 7-19-380

(7 - 19 - 204) - (2 - 5 - 204) = (5 - 13 - 176)

[46]5 - 13 - 176 = 143816

143816 x 74377 = 10,696,602,632

42 As part of the preparation of a paper about Raphael Hanover's book: לוחות העיבור.

43 See post face on p. 39 of the manuscript.

44 The text in bold does not appear in the printed edition; it must be an enthusiastic addition of the pupil.

45 The text is not very clear: we find the number of months preceding the given *molad*.

46 There is a mistake in this subtraction. We should find 5 – 14 – 176 = 144,896.

תחלוק ע״י 181440 ויוצא שארית 120312.[47]
תחלוק ע״י 235 ויוצא 511 ושארית 227.

וישארו לך רכ״ז חדשים. תחלוק ע״י י״ב חדשים ותמצא י״ח שנים י״א חדשים. תוסיף ששה חדשים בעבור ששה עיבורים שהן בי״ח שנים, ישארו לך ה׳ חדשים שעברו לשנת י״ט למחזור תקי״ב ונמצא זה המולד שאמרנו לעיל הניתן לך, הוא יהיה המולד של אדר ראשון משנת י״ט למחזור תקי״ב לבריאת העולם ודי לחכימא.

It appears that the student was less skilful than the teacher; he indeed made two mistakes so that the example is not very useful.

The correct calculation is the following:
Molad = (7) – 19 – 380 = 176420
T = 176420 – 31524 = 144896
A = $[144896 \times 74377]_{181440}$ = 119552 = 508 x 235 + 172
The given *molad* is the *molad* of Elul 9666, the 14th year of the cycle 509.

Verification
The number of months preceding the *molad* of 9667 is INT [(9666 x 235 + 1) / 19] = 119553. The *molad* of Tishri 9667 is $[31524 + 119553 \times 39673]_{181440}$ = 34653.

The *molad* of Elul 9666 is then 34653 – 39673 = 176420.

However this short passage is an invaluable historic testimony proving that Hanover knew the number 74377 and had discovered the way of calculating the first occurrence of any *molad*. The method described in this paper[48] and which I believe to be original was already discovered 250 years ago by Raphael of Hanover. Therefore I propose to call this method of calculating the first occurrence of any given *molad* **the method of Hanover**, and to name the number 74377 "**Raphael Hanover's number**."

Additional Remarks
1. Since the completion of this article I edited Hanover's manuscripts, including ספר תכונת השמים הארוך, and I found at the end of the book that Hanover improved the procedure. Instead of our modern formula, Hanover constructed convenient tables.

2. Already in the first half of the fourteenth century, R' Isaac Israeli proposed a

47 The remainder of the division is in fact 170,312.
48 See also my book: *Hilkhot Kiddush ha-Hodesh al-pi ha-Rambam*, J. J. Ajdler, Sifriati 1966, pp. 644-648.

solution to this problem[49] but it was less elegant and more difficult. The solution was based on two tables: the first table, לוח ג', gives the *molad* of the first 1080 months of the Jewish era. The first *molad* of the table is 2 – 5 – 204 and the last *molad* is 3 – 6 – 204.

Indeed $[1080 \times (1-12-793)]_{181440} = 27000 = 25920 + 1080 = 1d + 1h$.

Thus after 1080 months the *molad* is 1d 1h up.

The second table, לוח ד', gives the *molad* at the beginning of the first 168 cycles of 1080 months. After each cycle the *molad* is 1d 1h up. After 168 cycles the final *molad* is again the initial *molad*. Indeed 168 x (1d 1h) = 175d = M7.

3. Hanover's discovery of the integer 74377 was therefore not such an achievement. Hanover had the merit to search for which number of months the *molad* 2 – 5 – 204 is 1 *helek* up and becomes 2 – 5 – 205. He probably used the method of Israeli.

In לוח ג' we must find the *molad* ending with 205 *halakim*. This *molad* occurs after 937 months; it is 1 – 9 – 205. Indeed $[31524 + 937 \times 39673]_{181440} = 9925 = 1-9-205$.

We must add 20h in order to find the *molad* 2 – 5 – 205.

In לוח ד' we see that after 68 cycles of 1080 months the initial *molad* is 20h up. Indeed, $[68 \times (1d + 1h)]_{7d} = 20h$. Thus after 68 x 1080 + 937 = 74377 months the initial *molad* 2 – 5 – 204 became 2 – 5 – 205.

It appears that the finding of Raphael's Hanover's number using Israeli's algorithm did not present a major difficulty. Hanover's main originality was to search after how many months the *molad* is 1 *helek* up, and then propose a simple and elegant solution by constructing a table that gives the number of months necessary to obtain an increase of the *molad* by different multiples of 1 *helek*.

49 See *Yessod Olam*, Ma'amar V, chap. 4 and at the end of the book לוח ג' ולוח ד'.