

COMMENTS

J. Jean Ajdler, A reaction to Eran Raviv's articles published in B.D.D. nos. 22, 26, 27 and 28 versus my paper published in B.D.D. 28.

Eran Raviv published a series of important papers about the Jewish calendar, and more specifically about the four gates' table and the table of 61 rows, which he succeeded to expand to 213 rows in relating to the 30 indexed *keviyot* instead of the 14 traditional *keviyot*. The present letter to the editor is in no way a form of contradiction; it is only a complement to his learned papers.

1. Eran Raviv adopted a surprising convention, one which badly requires an explanation and elucidation. I noted indeed that he wrote in the four papers that $Beharad = 2 * 25920 + 5 * 1080 + 204 = 57444$ *hal* instead of 31524 *hal* (see my article B.D.D. 28 page 7).

In his papers B.D.D. 22 p. 45, B.D.D. 26 pp. 53, 57 and 59, B.D.D. 27 p. 73 and in B.D.D. 28 pp. 117-121, 124, 126 and 127 he classified the limits of his four gates' table and his table of 213 rows, departing from the beginning of the week, on Saturday 6 p.m. or $(1) - 0 - 0$ until the end of the week, $(7) - 23 - 1079$. His first limit in the table of 213 rows $(1) - 0 - 409$ is identified with 26329. Similarly his first limit in the four gates' table, $0 - 18 - 0$ is identified with 19440. This seems really surprising. In order to understand the problem, let us recall that a moment of the Jewish week is represented by a triplet $D - H - h$. D is the begun day, H is the number of completed hours and h is the number of completed *halakim*. Thus $D - H - h$ designates a moment in the D th day of the week after that H hours and h *halakim* of this day elapsed. Toomer calls this in *Ptolemy's Almagest* p. 276 note 4, the inclusive notation. Now if we want to express this moment in *halakim*, then we want to find the number of *halakim* elapsed from the beginning of the week until the considered moment. It is $(D - 1) * 25920 + H * 1080 + h$. This is the exclusive notation.

In order to make the difference clear, we use the following convention:
 $Beharad = (2) - 5 - 204 = 31524$ is the inclusive notation, the begun day being put in brackets.

$Beharad = 1 - 5 - 204 = 31524$ is the exclusive notation, completed days, hours and *halakim*.

At the beginning of the week: $(1) - 0 - 0$ the number of elapsed *halakim* is 0.

At the end of the week: $(7) - 23 - 1079 = 6 - 23 - 1079 = 181439$ *hal*, the number of elapsed *halakim*. Therefore in the table of 61 rows (B.D.D. 28 pp. 25-26) the

Jewish week is scanned from (1) – 0 – 0 until 6 – 23 – 1079 or from 0 *hal* until 181439 *hal* and the first line is 409 *hal*.

Eran Raviv, in order to make the calculations more convenient, and avoid being obliged to subtract one day when calculating the *halakim* and to add a day when going back from *halakim* to the classical notation, decided to express the moment in *halakim* by the formula $D * 25920 + H * 1080 + h$. That means that he chose a fictitious epoch of the Jewish week, shifted backwards by one day and placed it on Friday 6 p.m. at (7) – 0 – 0.

Now when the week begins on (1) – 0 – 0, the span of time between the epoch and the beginning of the week is indeed 25920 and when it ends on (7) – 23 – 1079 the span of time between the fictitious epoch and the end of the week is 207359 *hal*.

This explanation must imperatively be given; it will allow the reader to solve the apparent contradiction between the papers of Eran Raviv and mine. The convention of Eran Raviv is probably more convenient, as soon as one is aware of its principle, but mine corresponds to the truth and satisfies certainly more the purist. The reader will choose the best solution. *In any case one must always work with the same convention, otherwise it would be catastrophic.*

2. In B.D.D. 26 pp. 51-52, Eran Raviv noted that, at the origin, the four gates were related to the four possible days of Rosh ha-Shanah and that Yehiel Borenstein transposed them to the four types of years. However when we read *Sefer ha-Ibbur* of R. Abraham bar Hiya [edition Filipowski, London 1851 pp. 63-64 (II, 9)] and *Yessod Olam* of R. Isaac ha-Yisraeli [edition Baer Goldberg, Berlin 1858 (IV, 10) p. 19b], we see that they first introduced four gates according indeed to the four possible days of Rosh ha-Shanah and then, afterwards, they introduced again four gates corresponding to the four types of years. It seems thus that in fact we have two sets of four crossed gates. Today we follow Borenstein, who put the emphasis on the four types of years. Note, however, that Joffe (*Korot Heshbon ha-Ibbur* 1931 p. 171) explicitly divided the table into four columns corresponding to the four possible days of *Rosh ha-Shanah* and still put the emphasis on the days.

3. Eran Raviv expanded the table of 61 rows into a table of 213 rows and this is a remarkable achievement. However, if I am not mistaken he did not calculate the frequency of the 30 indexed *keviyot*. Of course he could have done it easily with his program.

A short explanation of the program and an organization chart of it would have interested the reader. It would have removed a part of the mysterious character of the results.

I note that Eran Raviv did not exploit all the resources and information that we can draw from the expansion of the four gates' table. The four gates' table is much easier to understand and to master. It allows a deeper understanding of the calendar and its *moladot*.

I will show that the calculation of the frequency of the indexed *keviyot* is easier this way than through the table of 213 rows. It can be performed manually and can be understood. With the table of 213 rows, we lose any control and **must** rely on the results of the computer. Because of the space required, we have divided the four gates into four different pages.

4. Principle of the calculation of frequencies.

The period of the calendar contains 689472 years = $19 * 36288 = 3.8 * 181440$. We have thus 36288 cycles in this period, each year of rank k repeats itself 36288 times in the great period of the Jewish calendar. But the 181440 instants (in *halakim*) of the week don't repeat themselves uniformly as a *molad* of the 689472 years. Some repeat themselves four times and others only three times. On average, each instant of the week is 3.8 times the *molad* of a year.

The 181440 instants of the week contribute to 689472 years.

10 consecutive instants contribute to 38 years i.e. they are the *moladot* of 38 years of the great period. Let us examine the years of the four gates.

First gate: years 2, 5, 10, 13 and 16. Each group of 10 consecutive instants of the week contributes to 10 years.

The year of rank 2 through the end-digits 0 and 5
 5 through the end-digits 1 and 6
 10 through the end-digits 7 and 2
 13 through the end-digits 8 and 3
 16 through the end-digits 9 and 4

Each end-digit contributes for 1 year.

Second gate: years 1, 4, 9, 12, 15. Each group of 10 consecutive instants of the week contributes to 10 years.

The year of rank 1 through the end-digits 4 and 9
 4 through the end-digits 5 and 0
 9 through the end-digits 1 and 6
 12 through the end-digits 2 and 7
 15 through the end-digits 3 and 8

Each end-digit contributes for 1 year.

Third gate: years 3, 6, 8, 11, 14, 17, 19. Each group of 10 consecutive instants of the week contributes to 14 years.

The year of rank 3 through the end-digits 6 and 1
6 through the end-digits 7 and 2
8 through the end-digits 2 and 7
11 through the end-digits 3 and 8
14 through the end-digits 4 and 9
17 through the end-digits 5 and 0
19 through the end-digits 0 and 5

The end-digits 1, 3, 4, 6, 8, 9 contribute, each of them, for one year. The end-digits 0, 2, 5, 7 contribute, each of them, for 2 years. Hence $4 * 2 + 6 = 14$ years.

Fourth gate: years 7, 18. Each group of 10 consecutive instants of the week contributes to 4 years.

The year of rank 7 through the end-digits 6 and 1
18 through the end-digits 4 and 9

Each end-digit 1, 4, 6 and 9 contributes for 1 year.

Conclusion: Each group of 10 consecutive instants of the week contributes to 38 years.

This allows the calculation of the number of years corresponding to a span of time of the Jewish week, whether it contains a multiple of 10 consecutive instants or a remainder of a number of consecutive instants less than 10 *halakim*.

For example, **in the first gate**, the span of time between 113604 and 123119 (inclusive) has an extent of 9516 instants: 951 groups of ten consecutive instants corresponding to $10 * 951$ years and a remainder of 6 instants with end-digits 4, 5, 6, 7, 8 and 9 corresponding to 6 additional years. **In the third gate**, the span of time between 90335 and 103679 (inclusive) has an extent of 13345 instants: 1334 groups of ten consecutive instants corresponding to $14 * 1334$ years and a remainder of 5 instants with end-digits 5, 6, 7, 8 and 9 corresponding to 7 additional years. The end-digit 5 corresponds to a year of rank 17 and a year of rank 19, the end-digit 6 corresponds to a year of rank 3, the end-digit 7 corresponds to a year of rank 6 and a year of rank 8. The end-digit 8 corresponds to a year of rank 11 and the end-digit 9 corresponds to a year of rank 14. In all, 7 years. **In the third gate**, the span of time between 151691 and 155519 (inclusive) has an extent of 3829 instants: 383 groups of ten consecutive instants corresponding to $14 * 383$ years minus one instant, 155520, improperly included. The end-digit of this instant is 0; it corresponds to a year of rank 17 and to a year of rank 19. In all 2 years which were counted too much. The considered span of time contributes to $14 * 383 - 2$

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= 5360 years. **In the fourth gate**, the span of time between 139524 and 155519 (inclusive) has an extent of 15996 instants: 1600 groups of ten consecutive instants corresponding to $4 \cdot 1600$ years minus 4 instants that were improperly included. Their end-digits are 0, 1, 2 and 3. A year of rank 7 was thus counted too much. The contribution of this area is thus finally $4 \cdot 1600 - 1 = 6399$. With these four examples we covered all the possible difficulties that could be met in the evaluation of the number of years corresponding to any span of time of the Jewish week, for any of the four kinds of years of the Jewish cycle of 19 years.

In the following tables (four gates' table expanded according to Eran Raviv's principle) I adopted the numbering of Eran Raviv for the *dehiyot* (see B.D.D. 22 p, 40 and B.D.D. 27 p. 65). It is preferable to my numbering adopted in B.D.D. 28 table 7 p. 19.

5. Errata. Correction of a few misprints in my paper in B.D.D. 28.

p. 10 last line: **689473** instead of 689743.

p. 21, 3 lines from bottom: **9516 halakim** instead of moladot.

p. 22, 3rd and 4th lines: **2651 halakim** instead of 9516 moladot.

Addenda and Corrigenda to my paper in B.D.D. 29, table p. 37.

In the first horizontal row, the indication: (1900)¹

The exponent 1 refers to the following note: Tables of Moon and Sun, Jean Meeus, Kessel-Lo, 1962, pp.186–192.

(1900) indicates that in these tables, the coordinates and the corrections were calculated with regard to an epoch of 1 / 1 / 1900.

Expanded four gates' table.

First gate: years 2, 5, 10, 13 and 16.

Lower limit	Upper limit	Size	Postpone Days	<i>dehiya</i>	<i>keviya</i>	Number years	Total
(1)-0-0 0	(1)-9-203 9923	9924	1	1	בהג	10 * 992 + 4	9924
(1)-9-204 9924	1-23-1079 25919	15996	1	1	בשה	1599*10 +6	15996
(2)-0-0 25920	(2)-17-1079 45359	19440	0	0	בשה	1944*10	19440
(2)-18-0 45360	(2)-23-1079 51839	6480	1	2	גכה	648*10	6480
(3)-0-0 51840	(3)-9-203 61763	9924	0	0	גכה	992*10 +4	9924
(3)-9-204 61764	(3)-17-1079 71279	9516	2	4	הכז	951*10 +6	9516
(3)-18-0 71280	(3)-23-1079 77759	6480	2	3	הכז	648*10	6480
(4)-0-0 77760	(4)-23-1079 103679	25920	1	1	הכז	2592*10	25920
(5)-0-0 103680	(5)-9-203 113603	9924	0	0	הכז	992*10 +4	9924
(5)-9-204 113604	(5)-17-1079 123119	9516	0	0	השא	951*10 +6	9516
(5)-18-0 123120	(5)-23-1079 129599	6480	2	3	זחא	648*10	6480
(6)-0-0 129600	(6)-23-1079 155519	9924	1	1	זחא	992*10 +4	9924
(6)-9-204 139524	(6)-23-1079 155519	15996	1	1	זשג	1599*10 +6	15996
(7)-0-0 155520	(7)-17-1079 174959	19440	0	0	זשג	1944*10	19440
(7)-18-0 174960	(7)-23-1079 181439	6480	2	3	בהג	648*10	6480
Total number		181440				36288*5	181440

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Second gate: years 1, 4, 9, 12 and 15.

Lower limit	Upper limit	Size	Postpone Days	<i>dehiya</i>	<i>keviya</i>	Number years	Total
(1)-0-0 0	(1)-9-203 9923	9924	1	1	בחג	10 * 992 + 4	9924
(1)-9-204 9924	1-23-1079 25919	15996	1	1	בשה	1599*10 +6	15996
(2)-0-0 25920	(2)-15-588 42708	16789	0	0	בשה	1678*10 +9	16789
(2)-15-589 42709	(2)-17-1079 45359	2651	1	5	גכה	265*10 +1	2651
(2)-18-0 45360	(2)-23-1079 51839	6480	1	2	גכה	648*10	6480
(3)-0-0 51840	(3)-9-203 61763	9924	0	0	גכה	992*10 +4	9924
(3)-9-204 61764	(3)-17-1079 71279	9516	2	4	הכז	951*10 +6	9516
(3)-18-0 71280	(3)-23-1079 77759	6480	2	3	הכז	648*10	6480
(4)-0-0 77760	(4)-23-1079 103679	25920	1	1	הכז	2592*10	25920
(5)-0-0 103680	(5)-9-203 113603	9924	0	0	הכז	992*10 +4	9924
(5)-9-204 113604	(5)-17-1079 123119	9516	0	0	השא	951*10 +6	9516
(5)-18-0 123120	(5)-23-1079 129599	6480	2	3	זחא	648*10	6480
(6)-0-0 129600	(6)-0-407 130007	408	1	1	זחא	40*10 +8	408
(6)-0-408 130008	(6)-23-1079 155519	25512	1	1	זשג	2551*10 +2	25512
(7)-0-0 155520	(7)-17-1079 174959	19440	0	0	זשג	1944*10	19440
(7)-18-0 174960	(7)-23-1079 181439	6480	2	3	בחג	648*10	6480
Total number		181440				36288*5	181440

Third gate: years 3, 6, 8, 11, 14, 17 and 19

Lower limit	Upper limit	Size	Postpone Days	<i>dehiya</i>	<i>keviya</i>	Number years	Total
(1)-0-0 0	(1)-20-490 22090	22091	1	1	בחה	2209*14 +2	30928
(1)-20-491 22091	1-23-1079 25919	3829	1	1	בשז	383*14 - 2	5360
(2)-0-0 25920	(2)-17-1079 45359	19440	0	0	בשז	1944*14	27216
(2)-18-0 45360	(2)-23-1079 51839	6480	1	2	גכז	648*14	9072
(3)-0-0 51840	(3)-17-1079 71279	19440	0	0	גכז	1944*14	27216
(3)-18-0 71280	(3)-23-1079 77759	6480	2	3	החא	648*14	9072
(4)-0-0 77760	(4)-11-694 90334	12575	1	1	החא	1257*14 +7	17605
(4)-11-695 90335	(4)-23-1079 103679	13345	1	1	השג	1334*14 +7	18683
(5)-0-0 103680	(5)-17-1079 123119	19440	0	0	השג	1944*14	27216
(5)-18-0 123120	(5)-23-1079 129599	6480	2	3	זחג	648*14	9072
(6)-0-0 129600	(6)-20-490 151690	22091	1	1	זחג	2209*14 +2	30928
(6)-20-491 151691	(6)-23-1079 155519	3829	1	1	זשה	383*14 - 2	5360
(7)-0-0 155520	(7)-17-1079 174959	19440	0	0	זשה	1944*14	27216
(7)-18-0 174960	(7)-23-1079 181439	6480	2	3	בחה	648*14	9072
Total number		181440				36288*7	254016

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Fourth gate: years 7 and 18.

Lower limit	Upper limit	Size	Postpone Days	<i>dehiya</i>	<i>keviya</i>	Number years	Total
(1)-0-0 0	(1)-9-203 9923	9924	1	1	בחג	4 * 992 +1	3969
(1)-9-204 9924	1-23-1079 25919	15996	1	1	בשה	1599*4 +3	6399
(2)-0-0 25920	(2)-15-588 42708	16789	0	0	בשה	1679*4 -1	6715
(2)-15-589 42709	(2)-17-1079 51839	2651	1	5	גכה	265*4 +1	1061
(2)-18-0 45360	(2)-23-1079 51839	6480	1	2	גכה	648*4	2592
(3)-0-0 51840	(3)-9-203 61763	9924	0	0	גכה	992*4 +1	3969
(3)-9-204 61764	(3)-17-1079 71279	9516	2	4	הכז	951*4 +3	3807
(3)-18-0 71280	(3)-23-1079 77759	6480	2	3	הכז	648*4	2592
(4)-0-0 77760	(4)-23-1079 103679	25920	1	1	הכז	2592*4	10368
(5)-0-0 103680	(5)-9-203 113603	9924	0	0	הכז	992*4 +1	3969
(5)-9-204 113604	(5)-17-1079 123119	9516	0	0	השא	951*4 +3	3807
(5)-18-0 123120	(5)-23-1079 129599	6480	2	3	זחא	648*4	2592
(6)-0-0 129600	(6)-9-203 139523	9924	1	1	זחא	992*4 +1	3969
(6)-9-204 139524	(6)-23-1079 155519	15996	1	1	זשג	1599*4 +3	6399
(7)-0-0 155520	(7)-17-1079 174959	19440	0	0	זשג	1944*4	7776
(7)-18-0 174960	(7)-23-1079 181439	6480	2	3	בחג	648*4	2592
Total number		181440				36288*2	72576

Frequencies of the indexed *keviyot*

Ordinary years				Leap years			
<i>keviyah</i>	<i>dehiyah</i>	Number Of years	Percent %	<i>keviah</i>	<i>dehiyah</i>	Number of years	Percent %
בחג		39369	5.7100	בחה		40000	5.8015
בחג	1	23817	3.4544	בחה	1	30928	4.4858
בחג	3	15552	2.2556	בחה	3	9072	1.3158
בשה		81335	11.7967	בשז		32576	4.7248
בשה	0	42944	6.2285	בשז	0	27216	3.9474
בשה	1	38391	5.5682	בשז	1	5360	0.7774
גכה		43081	6.2484	גכז		36288	5.2632
גכה	0	23817	3.4544	גכז	0	27216	3.9474
גכה	2	15552	2.2556	גכז	2	9072	1.3158
גכה	5	3712	0.5384				
הכז		124416	18.0451				
הכז	0	23817	3.4544				
הכז	1	62208	9.0226				
הכז	3	15552	2.2556				
הכז	4	22839	3.3125				
				החא		26677	3.8692
				החא	1	17605	2.5534
				החא	3	9072	1.3158
השא		22839	3.3125	השג		45899	6.6571
השא	0	22839	3.3125	השג	0	27216	3.9474
				השג	1	18683	2.7098
זחג		29853	4.3298	זחה		40000	5.8015
זחג	1	14301	2.0742	זחה	1	30928	4.4858
זחג	3	15552	2.2556	זחה	3	9072	1.3158
זשה		94563	13.7153	זשה		32576	4.7248
זשה	0	46656	6.7669	זשה	0	27216	3.9474
זשה	1	47907	6.9484	זשה	1	5360	0.7774
Total		435456	63.1579 = 12/19			254016	36.8421 = 7/19

Frequency of the different *dehiyot*

<i>Dehiyot</i>	Ordinary year	% ordinary years	Leap years	% leap years	Total years	% total years
0	160073	36.7599	108864	42.8571	268937	39.0062
1	186624	42.8571	108864	42.8571	295488	42.8571
2	15552	3.5714	9072	3.5714	24624	3.5714
3	46656	10.7143	27216	10.7143	73872	10.7143
4	22839	5.2449			22839	3.3125
5	3712	0.8524			3712	0.5384
Total	435456	100	254016	100	689472	100

Note: % ordinary years *12/19 + % leap years *7/19 = % total years.

6. The theorem of Bayes and its application in the Jewish calendar.

Let E1, E2, E3 and E4 be four different events, of which one must necessarily materialize. We assume that each of these events results from either an event C1 or an event C2 of which one must necessarily materialize. By convention we name each of the events C1 and C2 a possible cause of the event Ei. We assume to know the probability of intervening of the causes Pr (C1) and Pr (C2), Pr (Ei / C1) and Pr (Ei / C2) the probability of realization of the event Ei if the cause C1 or C2 materializes. We have thus Pr (C1) + Pr (C2) = 1
 Pr (E1 /Cj) + Pr (E2 /Cj) + Pr (E3 /Cj) + Pr (E4 /Cj) = 1 where j can take the values 1 and 2.

The formula of Bayes allows calculating Pr (Cj / Ei), the probability, when Ei materializes, of the intervention of the cause Cj:

$$\Pr (C_j / E_i) = \frac{\Pr(C_j) \Pr(E_i/C_j)}{\Pr(C_1) \Pr(E_i/C_1) + \Pr(C_2) \Pr(E_i/C_2)}$$

Example. We consider two urns. Each of them contains a known number of balls of four different colors. A blindfolded man pulls a ball out of one of the urns. We ascertain that it is a white ball. We want to know the probability that this ball was pulled out of the first urn.

Now we transpose the former example. The urn 1 contains the Ordinary years and the urn 2 contains the Leap years. The balls can have four colors, white, black, red and green. They represent years beginning on a Monday, Tuesday, Thursday and Saturday.

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The events E1, E2, E3 and E4 represent a Jewish year beginning on **M**, **Tu**, **Th** and **S**.

The events (the causes) C1 and C2, pulling a ball out of urn 1 or urn 2 represent pulling an **Ordinary** year or a **Leap** year. The formula of Bayes will allow us to calculate the probability that a random year beginning on a Monday is an Ordinary year.

In the example of the urns $\Pr(C1) = \Pr(C2) = 0.5$. In the cases of the Jewish years

$$\Pr(C1) = \Pr(O) = 12/19 = 0.6316. \Pr(C2) = \Pr(L) = 7/19 = 0.3684.$$

In the box 1 of the ordinary years the probability to pull a ball of a certain color or the probability of the events E1, E2, E3 or E4 is:

$$\Pr(E1 / C1) = \Pr(M / O) = 120704 / 435456 = 0.2772$$

$$\Pr(E2 / C1) = \Pr(Tu / O) = 43081 / 435456 = 0.0989$$

$$\Pr(E3 / C1) = \Pr(Th / O) = 147255 / 435456 = 0.3382$$

$$\Pr(E4 / C1) = \Pr(S / O) = 124416 / 435456 = 0.2857$$

In the box 2 of the leap years the probability to pull a ball of a certain color or the probability of the events E1, E2, E3 or E4 is:

$$\Pr(E1 / C2) = \Pr(M / L) = 72576 / 254016 = 0.2857$$

$$\Pr(E2 / C2) = \Pr(Tu / L) = 36288 / 254016 = 0.1429$$

$$\Pr(E3 / C2) = \Pr(Th / L) = 72576 / 254016 = 0.2857$$

$$\Pr(E4 / C2) = \Pr(S / L) = 72576 / 254016 = 0.2857$$

The formula of Bayes gives now:

$$\Pr(C1 / Ei) = \frac{\Pr(C1) \Pr(Ei/C1)}{\Pr(C1) \Pr(Ei/C1) + \Pr(C2) \Pr(Ei/C2)} \text{ and } \Pr(C2 / Ei) = 1 - \Pr(C1 / Ei).$$

Event E: A Jewish year begins on Monday, E1 is realized. The probability that it is an ordinary year is $\Pr(C1 / E1) = \Pr(O / M) = N / D$

$$N = 0.6316 * 0.2772 = 0.1751 \text{ and } D = 0.6316 * 0.2772 + 0.3684 * 0.2857 = 0.2803$$

$$N/D = 0.6245. \Pr(C1 / E1) = \Pr(O / M) = 0.6245 \text{ and } \Pr(C2 / E1) = \Pr(L / M) = 0.3755.$$

Event E2: A Jewish year begins on Tuesday, E2 is realized. The probability that it is an ordinary year is $\Pr(C1 / E2) = \Pr(O / Tu) = N / D$

$$N = 0.6316 * 0.0989 = 0.0625 \text{ and } D = 0.6316 * 0.0989 + 0.3684 * 0.1429 = 0.1151$$

$$N/D = 0.5428. \Pr(C1 / E2) = \Pr(O / Tu) = 0.5428 \text{ and } \Pr(C2 / E2) = \Pr(L / Tu) = 0.4570.$$

Comments

Event E3: A Jewish year begins on Thursday, E3 is realized. The probability that it is an ordinary year is $\Pr(C1 / E3) = \Pr(O / Th) = N / D$

$$N = 0.6316 * 0.3382 = 0.2136 \text{ and } D = 0.6316 * 0.3382 + 0.3684 * 0.2857 = 0.3133.$$

$$N/D = 0.6699. \Pr(C1 / E3) = \Pr(O / Th) = 0.6699 \text{ and } \Pr(C2 / E3) = \Pr(L / Th) = 0.3301.$$

Event E4: A Jewish year begins on Saturday, E4 is realized. The probability that it is an ordinary year is $\Pr(C1 / E4) = \Pr(O / S) = N / D$

$$N = 0.6316 * 0.2857 = 0.1805 \text{ and } D = 0.6316 * 0.2857 + 0.3684 * 0.2857 = 0.2857.$$

$$N/D = 0.6316. \Pr(C1 / E4) = \Pr(O / S) = 0.6316 \text{ and } \Pr(C2 / E4) = \Pr(L / S) = 0.3684.$$

Let us consider the event E3.

$\Pr(C1 / E3) = \Pr(O / Th) = 0.6699$ according to the Formula of Bayes.

The probability that a random year is E3 and belongs to C1, i.e. that it is an ordinary year beginning on Thursday is given, according to the principle of the composed probabilities, by:

$$\Pr(C1.E3) = \Pr(E3) * \Pr(C1 / E3) = (219831 / 689472) * 0.6699 = 0.318 * 0.6699 = 0.2136.$$

$$\Pr(C1.E3) = \Pr(C1) * \Pr(E3 / C1) = (12 / 19) * 0.3382 = 0.2136.$$

Direct verification.

$$\Pr(C1 / E3) = \Pr(O / Th) = 147255 / (147255 + 72576) = 0.6699.$$

$$\Pr(C1.E3) = \Pr(O.Th) = 147255 / 689472 = 0.2136.$$

The theorem of Bayes is fundamental in the cases where it is not possible to make a direct calculation of $\Pr(C_j / E_i)$. This is not the case with the Jewish calendar. It was however important to show that our table of the different frequencies fulfils the theorem of Bayes.